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How Good is Gold? Recognition of The Golden Rectangle

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Abstract

The Golden Proportion is the place where a line is divided in such a way that the ratio of the length of the shorter segment to the longer segment is equal to the ratio of the longer segment to the length of the whole line. It has been claimed by artists, architects, and aestheticians that the Golden Section is the most aesthetically pleasing division of a line, and that the Golden Rectangle is the most aesthetically pleasing of all rectangles. Although there is experimental support for these claims, it is not unequivocal. Many studies have been on preference for the Golden Rectangle. It is possible to recognize something and not prefer it, so one could still be sensitive to the Golden Proportion without preferring it in comparisons. The aim of the current study was to test how good people were at recognizing a Golden Rectangle (as opposed to preferring a Golden Rectangle). Jay Hambidge's (1920) writings focused on the Greek design of the Golden Section that was included in art and architecture. From this, McCulloch (1923) became interested in whether such a division of an area was pleasant. He wrote his masters thesis in 1923 in which his experiments were "designed to discover whether a basic preference exists for dynamic (characteristic of organic life) and the intermediate symmetries, how this preference is affected by the ability to discover symmetry, by the repetition of the act of judging, and by the optical illusions involved (p. 3)." McCulloch (1965) asserted, "I happen to have spent two years in measuring man's ability to set an adjustable oblong to a preferred shape, because I did not believe that he did prefer the golden section or that he could recognize it. He does and he can! On repeated settings for the most pleasing form he comes to prefer it and can set for it. The same man who can only detect a difference of a twentieth in length, area, or volume sets it at 1 to 1.618 (McCulloch, 1965, p. 395-96)." The current study's purpose was to investigate this claim. The current study reported data on ten observers who participated in four experimental conditions. This study was designed to see if, with little training, people could naturally pick out Golden Rectangles. In the first experimental condition, the observers were shown a series of 33 rectangles of different widths. There were eight rectangles smaller than a Golden Rectangle, the smallest being 217 by 144 pixels, and 24 rectangles larger than a Golden Rectangle, the largest being 281 by 144 pixels. The Golden Rectangle presented was 233 by 144 pixels. The observers were asked if in each instance the presented rectangle was wider than a Golden Rectangle. In the second condition, to test for directional symmetry, the rectangles varied vertically and observers were asked if each instance was taller than a Golden Rectangle. As a baseline control, observers were given the same tasks but were asked to judge rectangles according to how they compared to a square. The results showed that the task of judging the rectangles, and even the squares, was fairly difficult. Some observers performed systematically, whereas others did not. Responding to the square conditions was much more systematic and less variable than the Golden Rectangle conditions. It was discovered that the scale of two pixels and as well as the task were very hard, but not impossible because Subject 2 and Subject 8 were able to do complete the task relatively well. Some participants showed very systematic results across all the different sized rectangles, and some did not. Most conditions for the majority of the observers were non-monotonic but once the data were binned into larger groups, many of the subjects showed smooth curves. This study failed to support McCulloch's claim that people can recognize the Golden Rectangle.

Keywords: Golden Rectangle, Golden Section, Golden Proportion, Golden Ratio, Warren McCulloch, recognition, preference

Introduction

History of Golden Ratio

The Golden Section is a ratio that, in respect to a line segment, is divided in such a way that the ratio of the length of the shorter segment (a) to the longer segment (b) is the same ratio of the longer segment (b) to the whole line (a+b), and is defined as $a/b = b/(a+b)$, where $(a+b) = 1$. The Golden Section is an irrational number of approximately 0.618. By adding 1 to the Golden Section one gets an estimated 1.618, which is known as ϕ (see Figure 1), this also happens if one divides 1 by .618 as well.

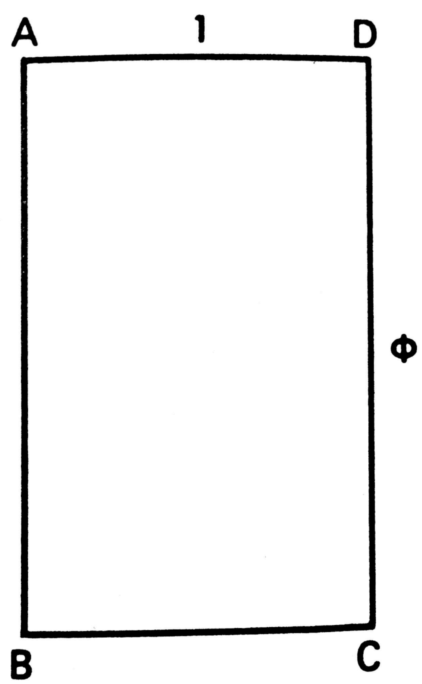

Figure 1. A Golden Rectangle is a rectangle with a ratio of 1:1.618. Adapted from "The Divine Proportion: A Study in Mathematical Beauty," by H. E. Huntley, 1970, p. 53. Copyright 1970 by Dover Publications, Inc.

The history of the Golden Section dates back to the ancient Egyptians. The "Golden Chamber" Papyrus of Rameses IV was buried in during 1149 B.C. had the dimensions of 16 x 16 x 10, which is "a golden right angled parallelepiped, defined by the lengths of its two adjoining sides, with dimensions of 1, ϕ, ϕ (Ghyka, 1977, p. 60).” It has been found that the “ancient Egyptians estimated ϕ within 0.5% accuracy, including ϕ into some designs of religious buildings (Green, 1995, p. 941).” The knowledge of the Golden Ratio made its way to ancient Greece, where Euclid, Proclus, and the Platonic Greek geometers discussed what they referred to as “the section” (Ghyka, 1977, p. 4).

“The line AB in [Figure 2a] is a perpendicular cutting the diagonal at a right angle at the point O, and BD is the square so created. BC is the line which creates a similar figure to the whole. One or unity should be considered as meaning a square. The number 2 means two squares, 3, three squares, and so on (Figure 2b). In [Figure 2a] we have the defined square BD, which is unity. The fraction .618 represents a shape similar to the original, or is its reciprocal. [Figure 2b] shows the reason for the name ‘rectangle of the whirling squares.’ 1, 2, 3, 4, 5, 6, etc., are the squares whirling around the pole O (Hambidge, 1920, p. 18).”

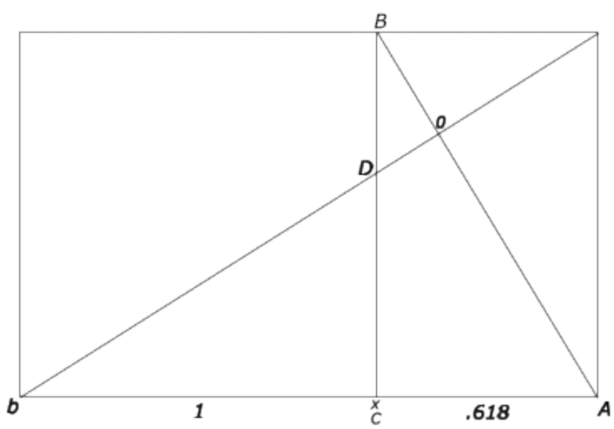

Figure 2a. Hambidge’s (1920) whirling rectangle. Adapted from “Dynamic Symmetry,” by J. Hambidge, 1920, p. 17. Copyright 1920 by Yale University Press.

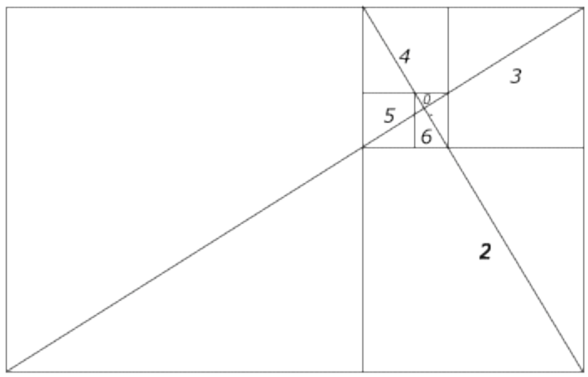

Figure 2b. Hambidge's (1920) whirling rectangle. Adapted from "Dynamic Symmetry," by J. Hambidge, 1920, p. 17. Copyright 1920 by Yale University Press.

The Roman architect Vitruvius focused on laws of proportion symmetry that defined beauty (Watts and Watts, 1986). Vitruvius as well as Leonardo Da Vinci found that the human body and its ratio of various parts have the proportion of Golden Section. Da Vinci's Vitruvian Man (Figure 3) shows that "in the human body every sort of proportion and proportionality can be found, produced at the beck of the all-Highest through the inner mysteries of nature (Paciolo, as cited by Livio, 2002, p.134)." The simplest example of this is the ratio of the total height of the body (1.618) to the height of the navel (1). "One can, in fact, state that if one measures this ratio for a great number of male and female bodies, the average ratio obtained will be 1.618 (Ghyka, 1997, p. 16)."

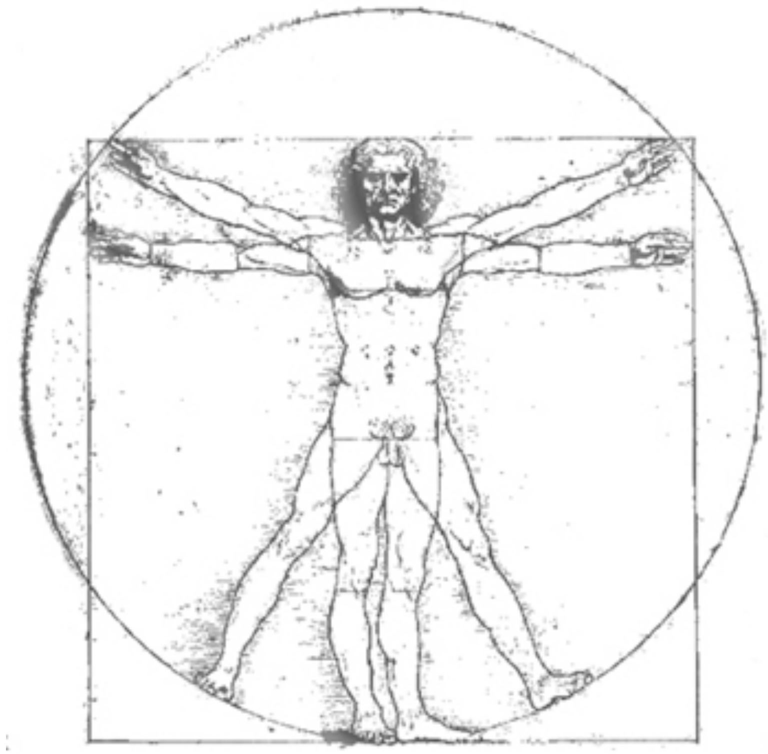

Figure 3. Leonardo Da Vinci's Vitruvian Man. Adapted from "Leonardo da Vinci," by M. Kemp, 2006, p. 97. Copyright 2006 by Oxford University Press.

Ancient Egyptians artists used square grids as a guide for their drawings of the human body, which were traditionally 18 squares tall. "The navel was placed along line eleven from the bottom, and that the ratio between eighteen (upper point) and eleven produces the value 1.636…, eighteen and eleven being, in fact, two terms of a Fibonacci-like series (Rossi, 2004, p. 81)." The same ratio can be found between "the height of the junction of the legs and the height of the junction of the armpits (Rossi, 2004, p. 81)." Rossi (2004) concluded that Golden Section-related geometrical figures and mathematical relationships can be found both in the art and architecture of ancient Egypt. "What appears clear is the modern psychological tendency to find the Golden Section everywhere (Rossi, 2004, p. 86)."

Leonardo da Pisa, commonly known as Fibonacci, was an Italian mathematician who solved a problem that involved the growth rate of rabbits based on idealized assumptions: the

Fibonacci sequence. The Fibonacci sequence begins with 0 and 1, and each successive number after that is the sum of the previous two numbers; the sequence is 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144,... If each of the numbers within the sequence is then divided into the number that precedes it, $\frac{1}{1},\frac{2}{1},\frac{3}{2},\frac{5}{3},\frac{8}{5},\frac{13}{8},\frac{21}{13},\frac{34}{21},\frac{55}{34},\frac{89}{55},\frac{144}{89},...$, respectively, then the ratios of successive numbers equal 1.000, 2.000, 1.500, 1.666, 1.600, 1.625, 1.615, 1.619, 1.617, 1.618, and 1.617, respectively. These ratios eventually converge to 1.618.

The Golden Ratio as a number value and the converging ratio of Fibonacci's sequence turned out to be the same number numerical value. This same number had turned up by two very different means, suggesting something extremely fundamental to people. Because Fibonacci's interesting series in arithmetic arrives at the same ratio of the Golden Ratio, people began to become interested in its mystic appeal.

Christopher D. Green (1995) explains that "the Golden Rectangle has a side length to side width ratio of 1:1.618, and because it has been claimed that the Golden Section is the most aesthetically pleasing division of a line, it has also been claimed that the Golden Rectangle is the most aesthetically pleasing of all rectangles (p. 937)."

The Golden Ratio has been found throughout history and permeates a broad range of cultures. Many Greek vases have ratios that correlate with the Golden Ratio. The Terracotta amphora vase that is attributed to the Amasis painter circa 550 B.C., currently on exhibit in the Metropolitan Museum of Art in New York, has the foot's width of a 2.472 rectangle that is described in the center of the 1.382 shape; the vase is composed of four whirling rectangles each with a ratio of .618 to the other rectangles (see Figure 4).

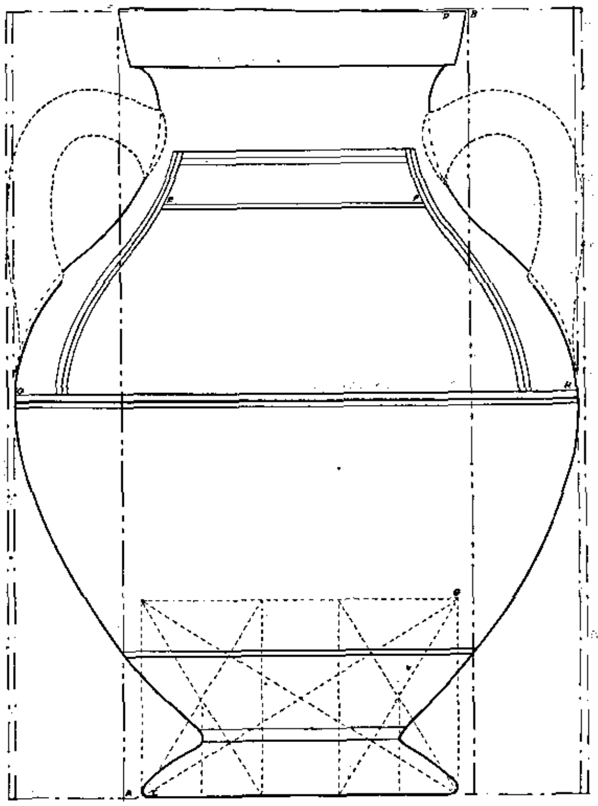

Figure 4. Black-figured Terracotta Amphora Vase 06.1021.69, 550 B.C. Metropolitan Museum of Art, New York, (measured and drawn by the Museum Staff). Adapted from "Dynamic Symmetry," by J. Hambidge, 1920, p. 98. Copyright 1920 by Yale University Press.

A similar Greek vase also at the Metropolitan Museum of Art, called the Terracotta skyphos vase, is attributed to the Theseus painter circa 500 B.C. and has an overall ratio of 1.854, or .618 multiplied by three (see Figure 5). The crosscuts of the vase are two whirling rectangles (see Figures 2a and 2b).

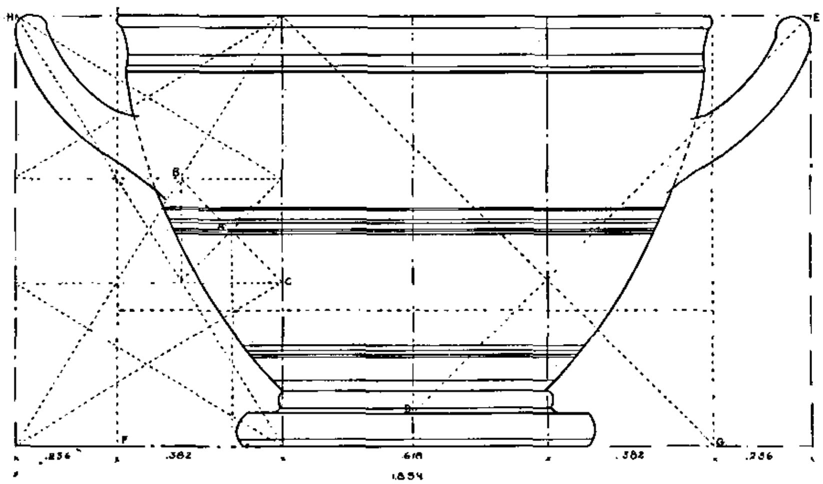

Figure 5. Terracotta Skyphos Vase. 500 B.C. Metropolitan Museum of Art, New York, (measured, drawn, and analyzed by L. D. Caskey). Adapted from "Dynamic Symmetry," by J. Hambidge, 1920, p. 106. Copyright 1920 by Yale University Press.

In 1965, Alexander Badawy, an Egyptian architect and Egyptologist suggested, "this proportion [the Golden Section] had been one of the main devices used by the Egyptians in the layout of their buildings…the Egyptians achieved the Golden Section by means of the Fibonacci Series…they adopted the ratio 8:5 (in which eight and five are numbers of the Fibonacci Series), which gives 1.6 as a result, as a good approximation for ϕ (Badawy, 1965 as cited by Rossi, 2004, p. 35)." Badawy successfully analyzed over 55 plans of Egyptian monuments from the Predynastic to the Ptolemaic period concluding, "a single set of rules was used throughout the entire history of Egyptian architecture…the Golden Section was among them (Badawy, 1965 as cited by Rossi, 2004, p. 43, 46)."

Past Studies

Gustav Fechner wrote about the Golden Section in 1871, and later in 1876 completed an experiment on the subject. Fechner "was the first experimentalist to study systematically the aesthetic properties of the Golden Section (Green, 1995, p. 942)."

Fechner used three methods of investigation: "the method of choice, in which subjects chose the item that they like and dislike the most; the method of production, in which subjects were asked to draw or create an object of a certain kind that had the features or proportions they find most agreeable and disagreeable; and the method of use, in which the experimenter examined preexisting objects and determined whether they conformed to his certain hypotheses about the determination of aesthetic pleasure (Green, 1995, p. 942)." Fechner (1876) presented each subject with a set of 10 white rectangles on a black table, with the proportions ranging from 1:1 to 2.5:1, all of which had equal areas. In the order of long-to-short ratio, the Golden Rectangle was ranked seventh, so that six rectangles had ratios lower than 1:1.618, and three had ratios higher than 1:1.618. The rectangles were randomly presented for each subject. Fechner (1876) had the subjects choose one or two rectangles that they liked the most. He then asked which rectangle(s) the subject found least pleasing. Fechner (1876) had a total of 347 responses, and of those, 35% chose the Golden Rectangle as most pleasing, while none chose the Golden Rectangle as the least pleasing. The Golden Rectangle was chosen as the most pleasing rectangle by 76% of the subjects. Fechner (1876) concluded from this data that the Golden Rectangle is the most aesthetically pleasing rectangle.

In 1894, Lightner Witmer replicated Fechner's (1876) experiment, but Witmer (1894) presented the rectangles serially to each subject. The results of his study found that a rectangle with a ratio of 1:1.651 was most preferred.

Charles Lalo (1908 as cited in Green, 1995, p. 262-64) also reproduced Fechner's (1876) study, in which he presented ten horizontal rectangles simultaneously to the participants and asked which they liked most. The Golden Rectangle was chosen by 30.3% of the subjects as the one they liked the most. Lalo (1908) also found that 18.3% preferred the two rectangles adjacent to the Golden Rectangle, 11.7% preferred squares, and 15.3% chose the 2.5:1 rectangle. Lalo's (1908) results found a similar trend to Fechner's (1876) study.

Out of 34 past studies (as cited in Green, 1995, Table 1, p. 962-64) on the preference of the Golden Rectangle, only six (Haines and Davies, 1904; Schiffman, 1969; Plug, 1976; Boselie, 1984a; Nakajima and Ohta, 1989; Davis and Jahnke, 1991) of the studies did not find a significant preference for the Golden Rectangle or a Golden shape (Green, 1995, Table 1, p. 962-64).

Berlyne (1970 as cited in Rossi, 2004, p. 79) tested Golden Section preferences cross-culturally, in which he compared the preferences of 33 Canadian high school girls and 44 Japanese high school girls. The Canadians' preferences peaked at the 1.5:1 rectangle, with 18% choosing it as most preferred. The Japanese subjects' preferences dropped off after the 1.5:1 rectangle. The Golden Rectangle was chosen by 9% of the Canadians and by 5% of the Japanese. Berlyne (1970 as cited in Rossi, 2004, p. 79) concluded, "that the Western preference for the Golden Section might be explained, at least in part, by the repeated exposure of the population to its diffused use in Western art since Egyptian and classical antiquity" (Berlyne, 1970 as cited in Rossi, 2004, p. 79).

Rossi (2004) summarized, "among Western populations there is a tendency, when asked to provide a series of positive or negative value-judgments on a discrete set of entities, to give a percentage of positive answers corresponding to a value very close to ϕ in comparison with the total number of answers (p. 78)."

Despite the ancient history and various artistic commitments to the Golden Ratio, the evidence for human sensitivity is still inconsistent. One aspect of the research traditions in experimental aesthetics is that most of the studies have been preference studies. It is possible that people could recognize a Golden Rectangle without preferring it. Therefore, we have chosen to try a recognition task in contrast to a preference task. The guiding questions were: Can people, with little or no training, recognize Golden Rectangles when they see them, or how close can they come? If they do not precisely pick out rectangles with the Golden Proportion, how close do they come, and how consistent are they?

Warren S. McCulloch

Warren S. McCulloch was a renowned American cybernetician and neurophysiologist who helped design computers, contributed to the cybernetics movement, and was prominently known for his work on brain theories. He believed that humans were sensitive to the Golden Proportion, but because it is an irrational number, this led him to think that the brain does not work quite like a computer does. McCulloch wrote his masters thesis at Columbia University in 1923 entitled "A Preference For Related Areas." Jay Hambidge's (1920) writings focused on the Greek design of the Golden Section that was either consciously or unconsciously included in art and architecture. This seemed to be proof enough that the ancient Greeks believed the Golden Section to be beautiful. From this, McCulloch (1923) became interested in whether such a division of an area was pleasant. His masters thesis, "had started out of incredulity of Hambidge's assertion that root rectangles and the Golden Section are aesthetically preferred by most people (McCulloch, 1974)." McCulloch (1923) conducted two experiments when he was at

Yale, which led him to further research. His experiments were "designed to discover whether a basic preference exists for dynamic (characteristic of organic life; nature-based) and the intermediate symmetries, how this preference is affected by the ability to discover symmetry, by the repetition of the act of judging, and by the optical illusions involved (McCulloch, 1923, p. 3)."

In a more recent publication by McCulloch (1965), he explained:

> I happen to have spent two years in measuring man's ability to set an adjustable oblong to a preferred shape, because I did not believe that he did prefer the golden section or that he could recognize it. He does and he can! On repeated settings for the most pleasing form he comes to prefer it and can set for it. The same man who can only detect a difference of a twentieth in length, area, or volume sets it at 1 to 1.618. So the aesthetic judgment bespeaks a precise knowledge of certain relations directly, not compounded of the simpler perceptibles. A sculptor or painter has sometimes told me he had added enough to a square so that the part he had added had the same shape as the whole. This example is pertinent here, for in this case we do have an adequate theory of the relations, namely ratio and proportion. But these apply only to the perceived object, not to its relation to the statement… The concept of a ratio must be embodied before the concept of a proportion can be conceived as the identity of the ratio… The golden section is a ratio that cannot be computed by any Turing machine without an infinite tape or in less than an infinite time. It is strictly incomprehensible. Yet it can be apprehended by finite automata, including us. Nor does it arise from any set of probabilities, or from a factor analysis of any data or correlation of observations, but as an insight – a guess, like every other hypothesis that is natural and simple enough to serve in science. (McCulloch, 1965, p. 395-96)

He conducted four experiments within his 1923 study; the first three experiments used four series of nine rectangular cards each that differed in width and had a horizontal line drawn through each rectangle at varying heights. The nine sizes of the rectangles differed starting at the fifth card by 5% narrower from cards four through one, and by 5% wider from cards six through nine, so that the percentages were as follows (compared to card five): rectangle one was narrower by 20%, rectangle two was narrower by 15%, rectangle three was narrower by 10%, rectangle four was narrower by 5%, rectangle six was wider by 5%, rectangle seven was wider by 10%, rectangle eight was wider by 15%, and rectangle nine was wider by 20%.

The first experiment was designed to discover whether a basic preference existed for dynamic and intermediate symmetries. This used the method of paired comparisons with the cards one meter from the participant on a table. The participant was asked to choose the rectangle he preferred out of each pair. He was then asked to pick one card from each series that had the top partial area similar to the entire area, but turned ninety degrees. Of the participants in this experiment, 20 were grouped as individuals trained in any formal or vocational manner in an artistic field, and the other group included 27 participants who were untrained in any artistic field. The data were graphed by the average preference and medians in terms of percentage, so that if all the subjects preferred a card, it was graphed at 100%, and if only half of the subjects preferred a card, it was graphed at 50%. “From these diagrams it is apparent that there is marked preference for card five, for it is preferred 80% of the times it is seen; that card six is always near it, for it is preferred 71.2% of the times seen…It is obvious that the trained subjects have a stronger and more consistent preference for the symmetrical card than have the untrained subjects (McCulloch, 1923, p. 6).” This experiment found that there is a basic preference for dynamic symmetry as compared to the intermediate symmetries, and for the intermediate as compared to static symmetry or to asymmetry.

The second experiment attempted to see how preference is affected by the ability to discover symmetry. This had the same materials as the first experiment, but used the method of selection so that the participant was asked to choose one rectangle from each of the four series. The participant was also asked to pick one card from each series that had the top partial area similar to the entire area, but turned 90 degrees. Of the participants in this experiment, 44 were grouped as individuals trained in any formal or vocational manner in an artistic field, and the other group included 56 participants who were untrained in any artistic field. The data were graphed by the percentage distribution of preferences by those who missed the symmetrical card when selecting for symmetry. “By the method of least squares, the correlation of the preferences for the symmetrical with the ability to find it, was plus 65.7%...the correlation of the number of preferences for a card with the number of times it was considered symmetrical in total was plus 94.3% (McCulloch, 1923, p. 7-8).” McCulloch found that the “ability to find is greater than the tendency to prefer the symmetrical; and that the tendency to prefer the symmetrical is strong (McCulloch, 1923, p. 8).” This experiment found that the preference for the dynamic and for the intermediate symmetries varied directly with the ability to discern symmetry.

The third experiment attempted to see how this preference is affected by the repetition of the act of judging. This used the same materials and method as the first experiment, but only two participants, one scientist and one musician, were used. Over 24 consecutive days the participants were asked to choose one rectangle each from the four series. The data were tabulate and “both subjects started with a random distribution of preference and ended by preferring the fifth card every time (McCulloch, 1923, p. 8).”

The fourth experiment attempted to see how this preference is affected by the optical illusions involved: the vertical-horizontal illusion and empty versus filled space. McCulloch (1923) made a machine that could be adjusted so that the participant could slide left and right for the vertical presentation and up and down for the horizontal presentation. Ten participants were asked to choose eight settings in each series that they found to be preferable. The experiment found that as the strength of the preference for the dynamic and for the intermediate symmetries increased, and the strength of optical illusion decreased rapidly, with the continued repetition of the act of aesthetic judgment. This experiment found that this preference affected approximately 1.9% of the symmetrical width by the optical illusion that verticals are greater than horizontals. "If it is affected at all by the optical illusion that filled space is greater than empty space, it is affected less than .3% of the symmetrical width (McCulloch, 1923, p. 2, 10)."

These findings constitute the foundation and the aesthetic validity of Jay Hambidge's (1920) theory of dynamic symmetry. McCulloch explained, "a man with a just noticeable difference of two per cent would set the card correctly to the third decimal point… Hambidge was correct! Man does live in a world of relations (McCulloch, 1974)."

Present Study

The current studies were designed to see if, with little training, people could naturally pick out rectangles whose sides were related to one another by the Golden Proportion. The traditional previous methods involved paired comparisons. These studies showed that the participants did not know what a Golden Rectangle was, but they did prefer a Golden Rectangle to a comparison rectangle. Thus, these studies were focused on preference of a Golden Rectangle, rather than an ability to recognize a Golden Rectangle.

We wanted to investigate McCulloch's (1965) claim that a man can "detect a difference of a twentieth in length, area, or volume [and] sets it at 1 to 1.618 (McCulloch, 1965, p. 395)." The aim of this study was to test how good the participants were at recognizing a Golden Rectangle, as opposed to preferring a Golden Rectangle.

Methods

The first Golden Rectangle condition was made up of 33 different sized rectangles that varied horizontally, shown at random, ten times each in a serial presentation. The second Golden

Rectangle condition was made up of 33 different sized rectangles that varied vertically, presented at random, ten times each in a serial presentation. The first square condition was made up of 20 different sized rectangles that varied horizontally, presented at random, ten times each in a serial presentation. The second square condition was made up of 20 different sized rectangles that varied vertically, shown at random, ten times each in a serial presentation. Because the design employed the method of constant stimuli, it was decided in advance that all of the participants saw all of the displays.

In the first experimental condition, observers were shown a series of rectangles of different widths and were asked if each instance was wider than a Golden Rectangle. In the second condition, to test for directional symmetry, the variation in the rectangles was in the vertical direction and observers were asked if each instance was taller than a Golden Rectangle. As a baseline control, observers were given the same tasks but, instead of Golden Rectangles, were asked if a given instance was wider (or taller) than a square.

Using the data collected, we estimated the differential threshold, which is the size of the difference required to say that something is definitely different from a standard, in this case, a square and a Golden Rectangle. The threshold is a theoretical sensory boundary for what is being noticed, so this threshold determined the ability to differentiate between a Golden Rectangle and any other rectangle. Within the data collected, the x-value where each graph's y-value equaled .5 was deemed to be the most likely threshold. This point demonstrated chance performance, where a subject was equally likely to judge a rectangle as wider than a Golden Rectangle and not wider than a Golden Rectangle.

All of the individual data was used to create psychometric curves in order to see the pattern of responding over the entire stimulus range. Psychometric curves are a standard way to present the collected data using the method of constant stimuli. The psychometric curve should have an "S" shape because stimuli are selected to obtain a series of "no" answers to the experimental question at one end, and "yes" answers at the opposite end. This then shows the value of where the area of uncertainty is; in which half of the answers are "no" and half are "yes". Key numbers were obtained from the individuals' psychometric curves. For many observers it was common for conditions to be unanalyzable, and therefore the data were grouped using a method called binning. After binning, smoother psychometric curves were drawn and characteristic numbers were determined from the graphs.

Displays and Their Presentation

The four conditions were created using a Psychtoolbox program adapted from MullerLyer (2008) model experiment in Matlab. The program used was completed by William M. Mace (2013).

In the first Golden Rectangle condition, participants were asked to judge if the rectangle shown was wider than a Golden Rectangle, pressing the "Y" key for yes, the rectangle shown was wider than a Golden Rectangle, and the "N" key for no, the rectangle shown was not wider than a Golden Rectangle. In the second Golden Rectangle condition, the participants were asked to judge if the rectangle shown was taller than a Golden Rectangle, pressing the "Y" key for yes, the rectangle shown was taller than a Golden Rectangle, and the "N" key for no, the rectangle shown was not taller than a Golden Rectangle. In the first square condition, participants were asked to judge if the rectangle shown was wider than a square, pressing the "Y" key for yes, the rectangle shown was wider than a square and the "N" key for no, the rectangle shown was not wider than a square. In the second square condition, participants were asked to judge if the rectangle shown was taller than a square, pressing the "Y" key for yes, the rectangle shown was taller than a square, and the “N” key for no, the rectangle shown was not taller than a square. Each condition was judged by all 10 participants.

The first Golden Rectangle condition included one Golden Rectangle with the dimensions of 233 by 144 pixels (see Figure 6). The first Golden Rectangle condition varied horizontally, with each rectangle differing in width by two pixels. The skinniest rectangle used was 217 by 144 pixels (see Figure 7), and the widest rectangle used was 281 by 144 pixels (see Figure 8).

Figure 6. Target horizontal Golden Rectangle in the first Golden Rectangle condition, 233 x 144 pixels.

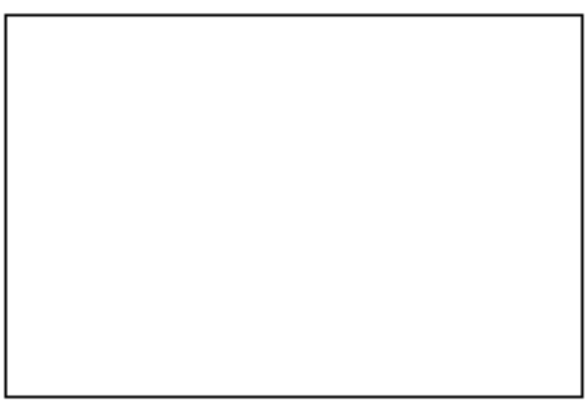

Figure 7. Skinniest horizontal rectangle presented in the first Golden Rectangle condition, 217 x 144 pixels.

Figure 8. Widest horizontal rectangle presented in the first Golden Rectangle condition, 281 x 144 pixels.

The second Golden Rectangle condition included one Golden Rectangle with the dimensions of 144 by 233 pixels (see Figure 9). The second Golden Rectangle condition varied vertically, with each rectangle differing in height by two pixels. The shortest rectangle used was 144 by 217 pixels (see Figure 10), and the tallest rectangle used was 144 by 281 pixels (see Figure 11).

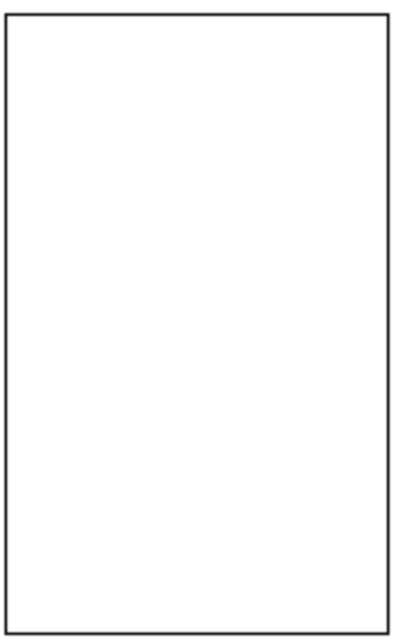

Figure 9. Target vertical Golden Rectangle in the second Golden Rectangle condition, 144 x 233 pixels.

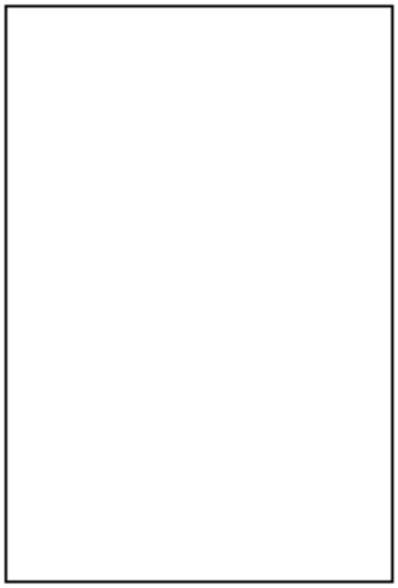

Figure 10. Shortest vertical rectangle presented in the second Golden Rectangle condition, 144 x 217 pixels.

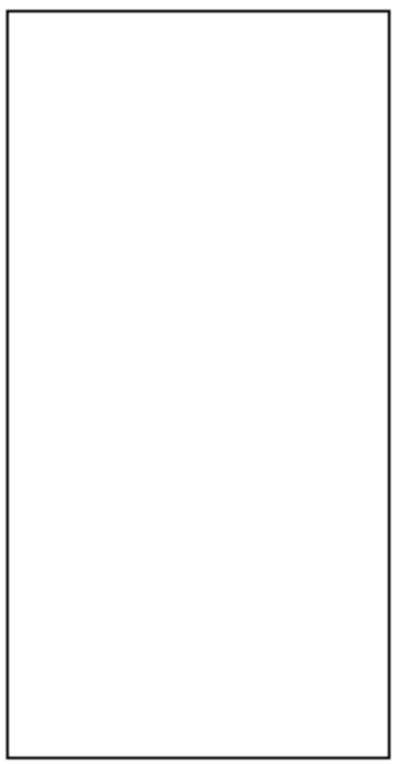

Figure 11. Tallest vertical rectangle presented in the second Golden Rectangle condition, 144 x 281 pixels.

The first square condition included one square with the dimensions of 144 by 144 pixels. The first square condition varied horizontally, with each rectangle differing in width by two pixels. The skinniest rectangle used was 124 by 144 pixels; the widest rectangle used was 162 by 144 pixels.

The second square condition included one square with the dimensions of 144 by 144 pixels. The second square condition varied vertically, with each rectangle differing in height by two pixels. The shortest rectangle used was 144 by 124 pixels; the tallest rectangle used was 144 by 162 pixels.

Participants

Ten Trinity College students participated in the experiment. Two of the participants were male, and eight were female. Three had never taken an art class at Trinity College, four had taken one art class at Trinity College, two had taken multiple art classes at Trinity, and one person was an art history major (see Appendix A).

Procedure

The methodology was approved by the Trinity College Institutional Review Board. The four conditions were created using a Psychtoolbox program adapted from MullerLyer (2008) model experiment in Matlab.

The participants began each of the conditions with brief instructions that flashed on the screen. By pressing any key, the next screen appeared instructing the participant to begin with three trials in order to establish a basic understanding of the task. After the participants completed the three practice trials the experiment began. Each Golden Rectangle experiment contained 330 trials. On each trial, one of the rectangles was shown at a random location on the screen. Each of the 33 different rectangles was shown ten times. The 330 presentations were shown in random order. Each square experiment contained 200 trials. On each trial, one of the rectangles was shown at a random location on the screen. Each of the 20 rectangles was shown ten times. The 200 presentations were shown in random order. When all the trials were judged a message appeared informing the participants, “That is it! Thank you for participating!”

The first Golden Rectangle practice trial began with the instructions, “Press any key to see the first example of a Golden Rectangle. Press a key again to see the second example, then a third. The experiment will begin after that.” These three practice trials were the only opportunity the participants had to see correctly identified Golden Rectangles because they received no feedback during the experiment. Rather than asking a “yes” or “no” question for each trial, participants were told, “In this experiment you are asked to judge when a rectangle is wider than a Golden Rectangle. Press ‘y’ if the rectangle is wider than a Golden Rectangle. Press ‘n’ is the rectangle is not wider than a Golden Rectangle. You will begin with 3 training trials.” The question was put this way in order to allow the data to be presented in the standard way for the method of constant stimuli.

The second Golden Rectangle trial began with the same instructions as the first Golden Rectangle trial, but asked to judge if the rectangle was taller than a Golden Rectangle.

The first square trial began with the instructions, “In this experiment you are asked to judge when a rectangle is wider than a square. Press ‘y’ if the rectangle is wider than a square. Press ‘n’ is the rectangle is not wider than a square. You will begin with 3 training trials.”

The second square trial began with the same instructions as the first square trial, but asked to judge if the rectangle was taller than a square.

Results

Each participant's estimate of the size of a Golden Rectangle or a square is shown in Table 1. The pattern of responding was so variable across participants that much of the data will be reported person by person.

For many observers it was common for the data conditions to be unanalyzable due to the fact that the rectangles differed by only two pixels each, so the data were grouped into bins. After binning, smoother psychometric curves could be created and characteristic numbers were determined from the graphs. The method presumed that if there was a rectangle size to which people said "no" every time, then all the sizes smaller than that would be all "no's". Once there was a size that had a few "yes" answers, it would be expected that the "yes" answers would either remain about the same or increase as the rectangles increased in size; therefore the curve of these results would be monotonic. At the size difference of two pixels, it was found that there were many reversals where people changed direction in the number of "yes" responses they gave. This showed that people could not tell the difference between the rectangles in those size ranges. Where there were many reversals over some range of sizes the $y = .5$ point was difficult to determine.

Table 1

Subject's p('yes') = .5 and standard deviations (SD).

Observer #	Horizontal Golden Rectangle 1	Horizontal Golden Rectangle 1	Vertical Golden Rectangle 2	Vertical Golden Rectangle 2	Horizontal Square 1	Horizontal Square 1	Vertical Square 2	Vertical Square 2
	p('yes')=.5	SD	p('yes')=.5	SD	p('yes')=.5	SD	p('yes')=.5	SD
1	236.83	25.89	231.91	21.17	148.00	7.88	140.71	10.11
2	246.33	19.72	261.62	17.44	154.67	6.30	146.40	6.25
3	256.71	18.83	252.00	17.44	150.77	11.86	153.78	7.73
4	259.33	19.36	237.25	21.62	N/A	N/A	153.07	14.45
5	270.00	29.96	261.00	19.61	150.59	6.58	143.58	7.14
6	249.67	14.59	243.50	18.51	149.00	9.28	144.80	10.66
7	N/A	31.12	242.76	26.22	137.90	9.41	N/A	16.80
8	259.89	11.53	251.00	10.63	157.00	2.97	148.00	3.74
9	249.42	13.71	256.00	21.02	146.53	9.22	156.00	8.46
10	261.62	18.08	271.60	22.65	N/A	N/A	152.73	7.36

Note. N/A means that the scores were incalculable due to inconsistencies in the data.

Table 2

Percentage differences from 233 pixels of each value.

Pixel	Pixel/144	Ratio/Golden		Percentage Difference from 233/144 or 144/233
228	1.583	0.979	0.021	2.15%
229	1.590	0.983	0.017	1.72%
231	1.604	0.991	0.009	0.86%
233	1.618	1	0.000	0.00%
235	1.6319	1.009	0.009	0.86%
237	1.646	1.017	0.017	1.72%
238	1.653	1.021	0.021	2.15%

Note. The values in red signify a 2 percent difference range.

Table 1 shows the probability that an individual responded “no” to half of the answers and “yes” to the other half. The results of the data were not conclusive, but two of the subjects’ rectangle judgments had small similarities compared to the square judgments for each respective orientation. Subject 4’s and Subject 8’s horizontal Golden Rectangle judgments were overestimated by the same amount (26 pixels). Both subjects’ judgments for all conditions were larger than the target rectangles, but the vertical orientations were smaller than the horizontal orientations judgments. This could be because both subjects were affected by the vertical-horizontal illusion, in which the horizontal judgments were wider in order to compensate for the taller perceived verticals.

All subjects’ judgments for the horizontal Golden Rectangle were overestimated. All subjects but one overestimated their judgments for the vertical Golden Rectangle. All subjects but one overestimated their judgments for the horizontal square. All subjects but two overestimated their judgments for the vertical square. Out of the 40 probabilities that half of the judgments were “yes” for the answers at .5, only four judgments were not overestimated.

At the original level of resolution at two pixels, there was a great deal of variability. The people with the greatest amount of variability were Subject 4 and Subject 7 (see Table 1).

Table 2 shows the percentage difference from 233 pixels of each other value. In order to get close to 2%, intermediate values of 228 and 238, which were not included in the study, were inserted. Roughly, 228 and 238 are boundaries of the 2% range. McCulloch claimed “a man with a just noticeable difference of two percent would set the card correctly to the third decimal point (McCulloch, 1974),” which was calculated for this study to be between 228 pixels and 238 pixels. Only one subject was able to detect the difference of the rectangles with a 2% difference, Subject 1, but their large standard deviation may account for the subject’s scores to be within the

2% difference range. Subject 4’s vertical Golden Rectangle judgment was the only other judgment that was within the 2% difference range.

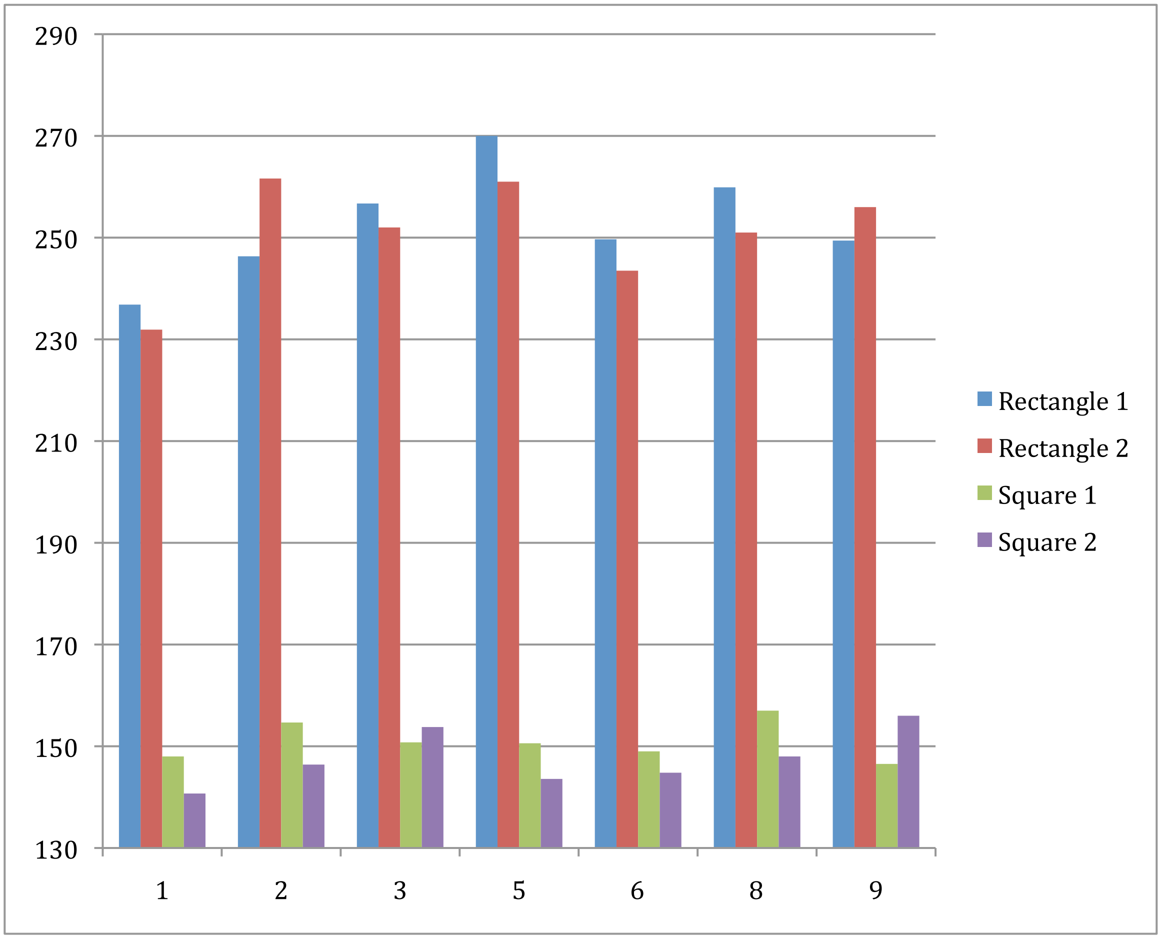

Figure 12. Individual’s x-values where the graph’s y-values = .5. The individual subject numbers are along the x-axis. Each Subject’s x-values where the graph’s y-values = .5 are along the y-axis.

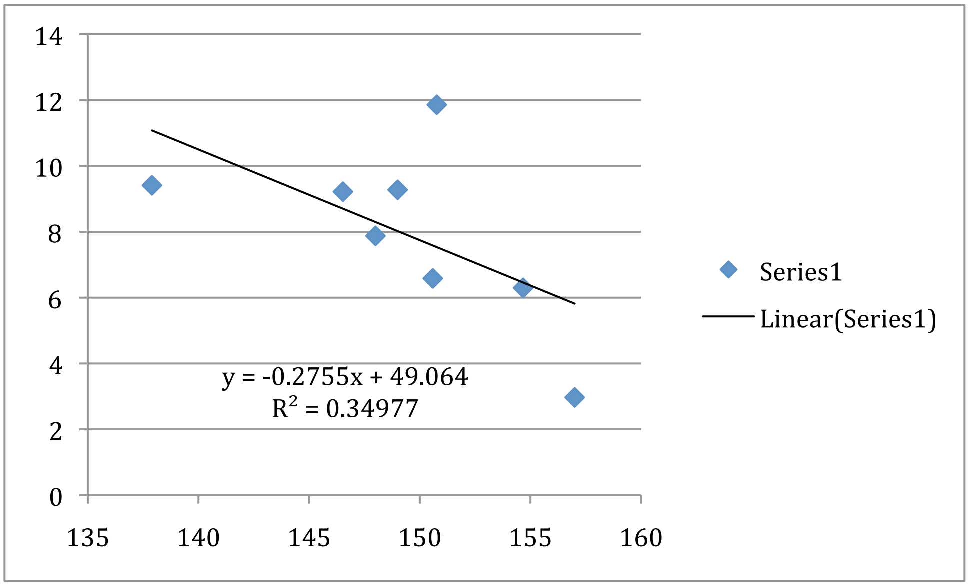

Figure 13. R^2 = .35, indicating a moderate negative correlation.

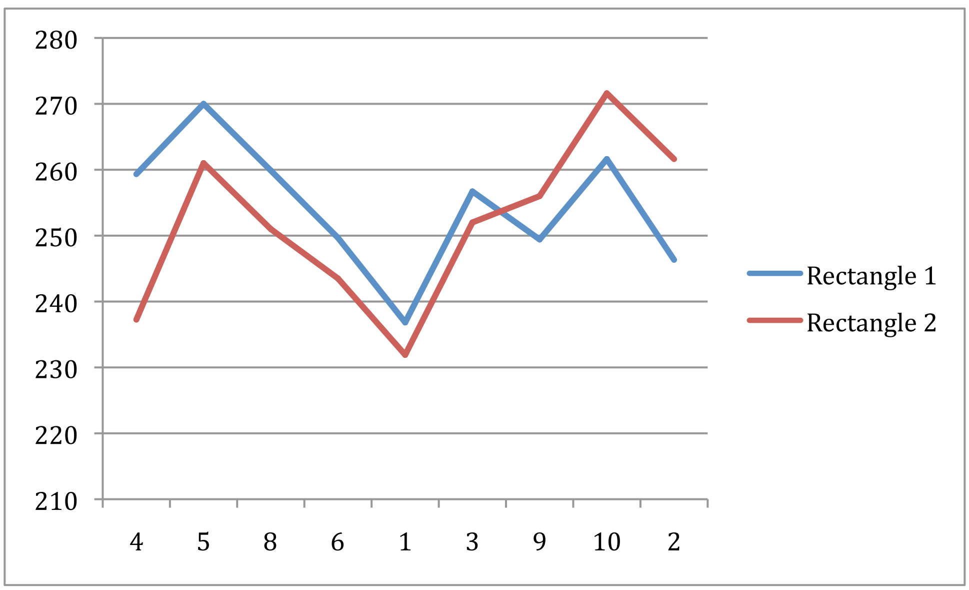

Figure 14. Individual's x-values where the graph's y-values = .5 for both horizontal and vertical Golden Rectangles. The individual subject numbers are along the x-axis. Each Subject's x-values where the graph's y-values = .5 are along the y-axis. The points are sorted by order of difference between rectangles. Rectangle 1 is greater than rectangle 2 for six participants. Rectangle 2 is greater than rectangle 1 for three participants.

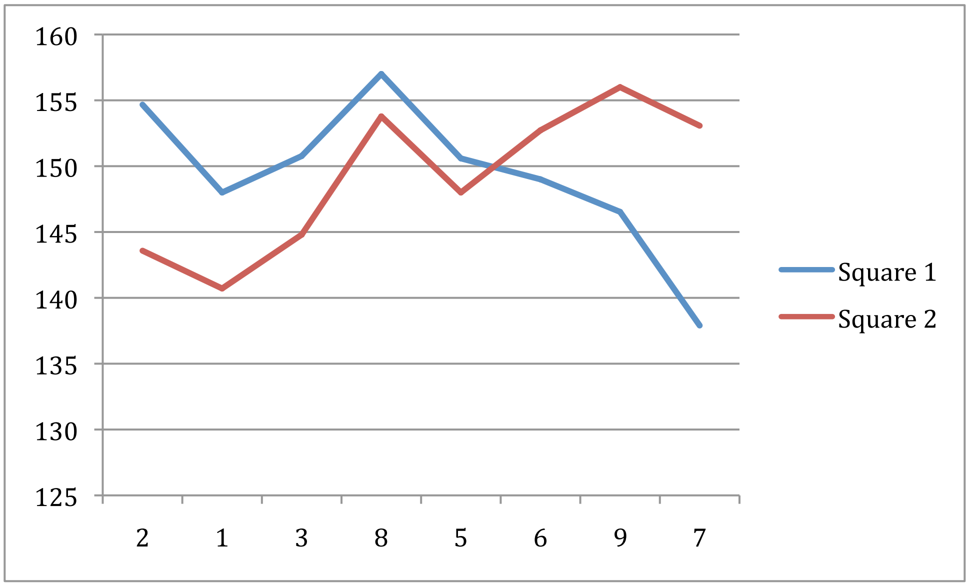

Figure 15. Individual's x-values where the graph's y-values = .5 for both horizontal and vertical squares. The individual subject numbers are along the x-axis. Each Subject's x-values where the graph's y-values = .5 are along the y-axis. Square 1 is greater than square 2 for five participants. Square 2 is greater than square 1 for three participants.

Figure 12 shows the individual x-values where the graph's y-values equaled .5. The individual subject numbers are along the x-axis. Each subject's x-values where the graph's y-values equaled .5 are along the y-axis. Figure 12 shows the individual comparisons to the other subjects.

Figure 13 shows the index of association that was used to interpret the effect size. The R^2 found was .35, which indicates a moderate negative correlation.

Figure 14 shows the individual x-values where the graph's y-values equaled .5 for both the horizontal and vertical Golden Rectangles. The individual subject numbers were plotted along the x-axis, while each subject's x-values where the graph's y-values equaled .5 were plotted along the y-axis.

Figure 15 shows the individual x-values where the graph's y-values equaled .5 for both the horizontal and vertical squares. The individual subject numbers were plotted along the x-axis, while each subject's x-values where the graph's y-values equaled .5 were plotted along the y-axis.

Subject 1

The y-value at .5 for the horizontal Golden Rectangle presentation was 236.83 (*SD* = 25.89; see Figures 16a for an example of non-binned data and 16b for an example of binned data). This judgment was within the 2% difference range (see Table 2). The y-value at .5 for the vertical Golden Rectangle presentation was 231.91 (*SD* = 21.17). This judgment was within the 2% difference range (see Table 2). The y-value at .5 for the horizontal square presentation was 148.00 (*SD* = 7.88). The y-value at .5 for the vertical square presentation was 140.71 (*SD* = 10.11). Subject 1 had similar values for both the horizontal *(p('yes') at .5* = 236.83) and vertical (*p('yes') at .5* = 231.91) Golden Rectangles, but this pattern did not hold for the horizontal *(p('yes') at .5* = 148.00) and vertical *(p('yes') at .5* = 140.71) squares (see Table 1, Figures 12, 14, and 15).

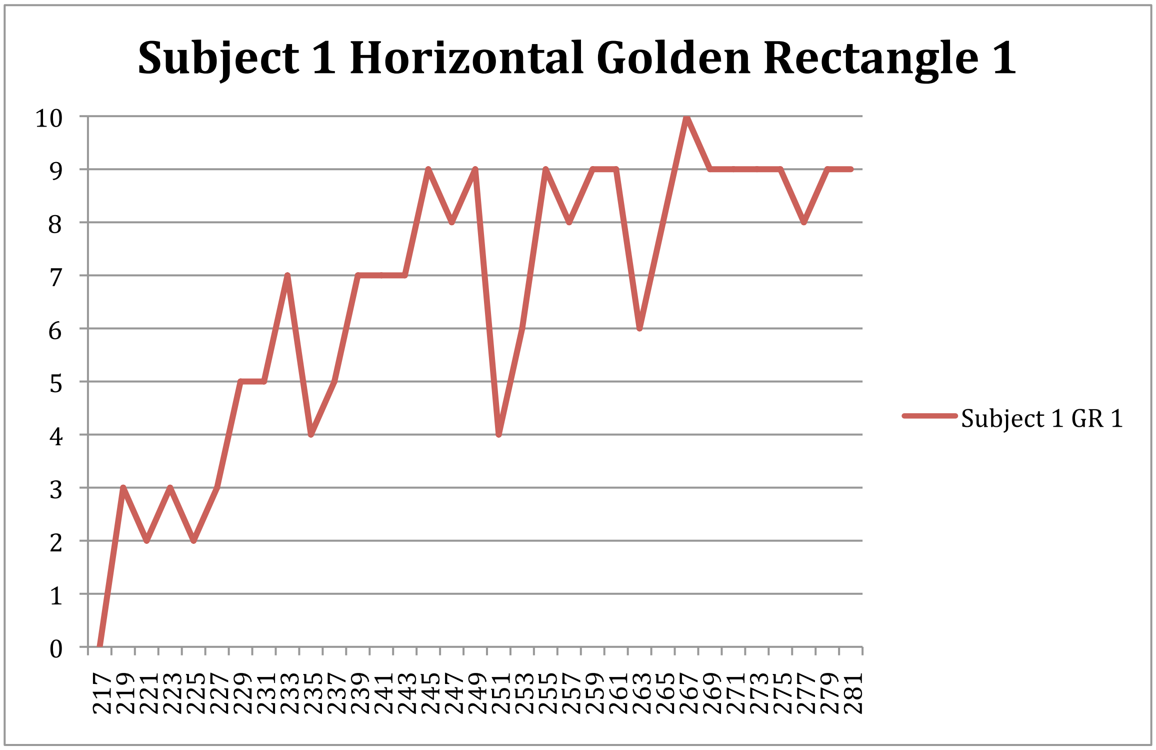

Figure 16a. Subject 1 horizontal presentation Golden Rectangle 1 results, non-binned.

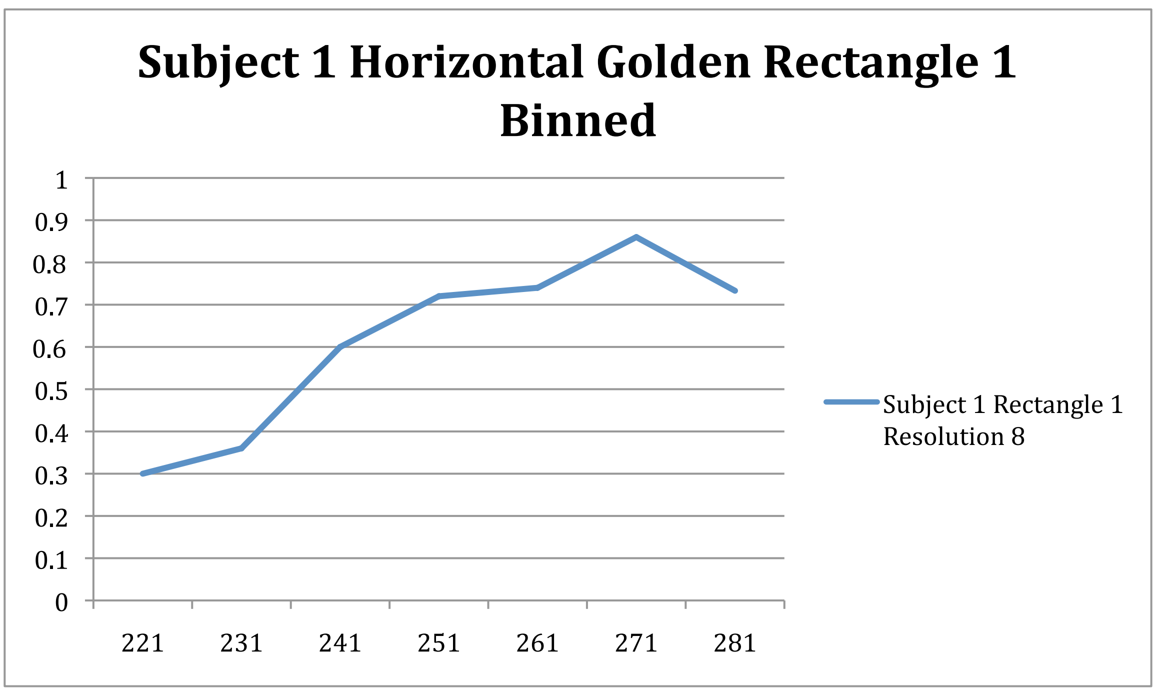

Figure 16b. Subject 1 horizontal presentation Golden Rectangle 1 results binned data.

Subject 2

The y-value at .5 for the horizontal Golden Rectangle presentation was 246.33 (*SD* = 19.72). The y-value at .5 for the vertical Golden Rectangle presentation was 261.62 (*SD* = 17.44). The y-value at .5 for the horizontal square presentation was 154.67 (*SD* = 6.30). The y-value at .5 for the vertical square presentation was 146.40 (*SD* = 6.25; see Figures 12, 14, and 15).

Subject 4

Subject 4's y-value at .5 for the horizontal Golden Rectangle presentation was 259.33 (*SD* =19.36). The y-value at .5 for the vertical Golden Rectangle presentation was 237.25 (*SD* = 21.62). The y-value at 5 for the horizontal square presentation was incalculable due to data inconsistencies (see Figure 17 for an example of unscorable data that could not be binned for a smoother graph). The y-value at .5 for the vertical square presentation was 153.07 (*SD* = 14.45).

All of Subject 4's judgments were overestimated; the largest judgment was for the horizontal Golden Rectangle presentation of 26 pixels (or 13 presented rectangles) wider than the target Golden Rectangle. The horizontal square judgment was incalculable so that there is no baseline comparison available. The vertical Golden Rectangle was judged four pixels (or two presented rectangles) taller than the target Golden Rectangle, and the vertical square was judged nine pixels taller than the target square. These results show that Subject 4's judgments were skewed due to the vertical-horizontal illusion, so that the horizontal judgments were wider in order to compensate for the perceived taller verticals (see Figures 12, 14, and 15).

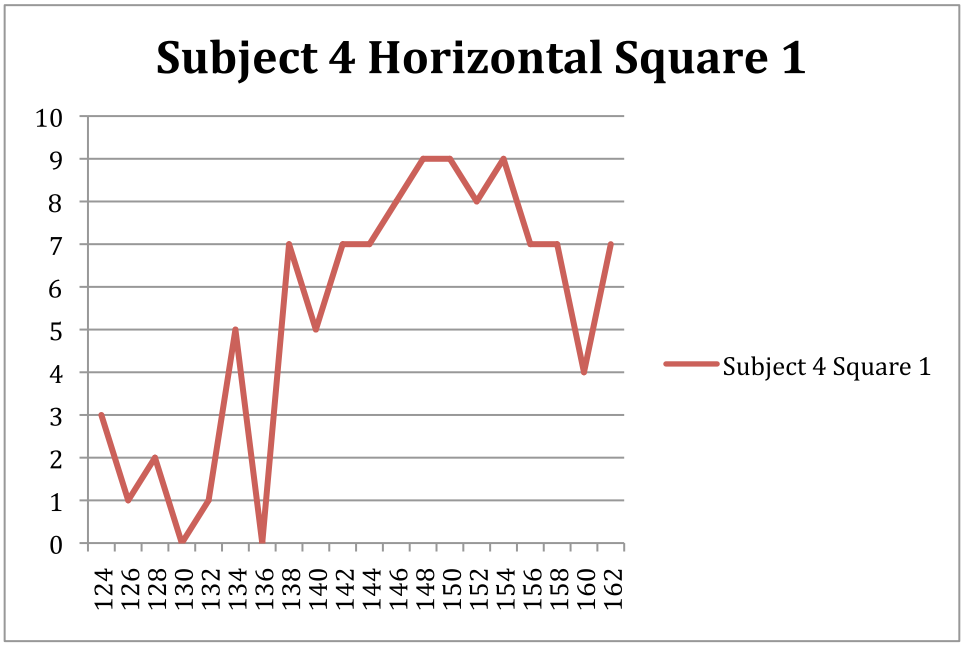

Figure 17. Subject 4 horizontal presentation square 1 results, which were unscorable and unable to be binned.

Subject 7

Subject 7's y-value at 5 for the horizontal Golden Rectangle 1 presentation was incalculable due to data inconsistencies (*SD* = 31.12; see Figure 18 for an example of unscorable data that could not be binned for a smoother graph). The y-value at .5 for the vertical Golden Rectangle presentation was 242.76 (*SD* = 26.22). The y-value at .5 for the horizontal square presentation was 137.90 (*SD* = 9.41). The y-value at .5 for the vertical square presentation was incalculable due to data inconsistencies (*SD* = 16.80). For the cases that were supposed to be obviously too skinny to be squares, which were designed to elicit all "no's" from an individual, were judged as "yes's", so that the presented rectangle was too wide to be a square four times for each of the smallest values. At the point where the rectangles should be obviously too wide, the individual said that it was too wide only five times, when it should have been all ten times. There were cases at both extremes, which were nearly the same for Subject 7 (See Figures 12, 14, 15, and 18).

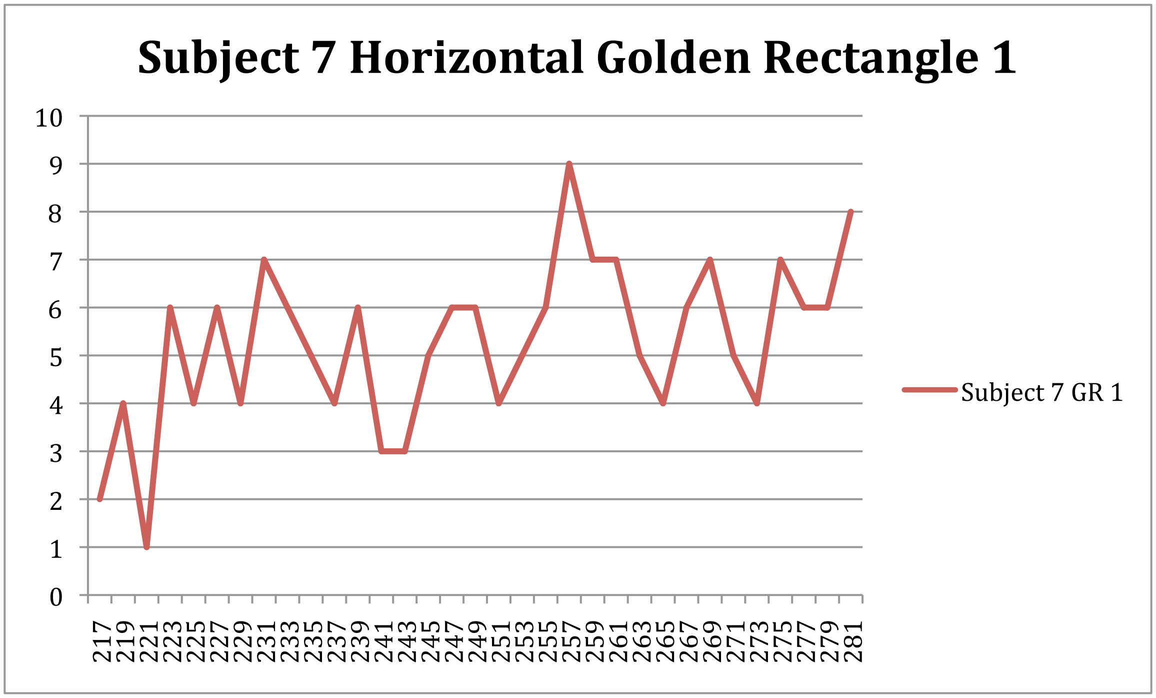

Figure 18. Subject 7 horizontal presentation Golden Rectangle 1 results, which were unscorable and unable to be binned.

Subject 8

The person with the least amount of variability was Subject 8. The y-value at .5 for the horizontal Golden Rectangle presentation was 259.89 (*SD* =11.53). The y-value at .5 for the vertical Golden Rectangle presentation was 251.00 (*SD* = 10.63). The y-value at .5 for the horizontal square presentation was 157.00 (*SD* = 2.97; see Figure 19). The y-value at .5 for the vertical square presentation was 148.00 (*SD* = 3.74; see Figure 20). Subject 8 also seemed to be affected by the vertical-horizontal illusion. All of Subject 8's judgments were overestimated; the largest judgment was for the horizontal Golden Rectangle presentation of 26 pixels (or 13 presented rectangles) wider than the target Golden Rectangle. The horizontal square judgment was 13 pixels wider than the target square. The judgments for the vertical presentations were not as skewed as the horizontal judgments, but were still overestimated. The vertical Golden Rectangle presentation was 18 pixels (or nine presented rectangles) taller than the target Golden Rectangle, and the vertical square presentation was four pixels (or two presented rectangles) taller than the target square. The standard deviations on all of Subject 8's judgments were the lowest of all the participants, meaning that Subject 8 had the least amount of variability and were very systematic in their judgments (see Table 1, Figures 12, 14, and 15).

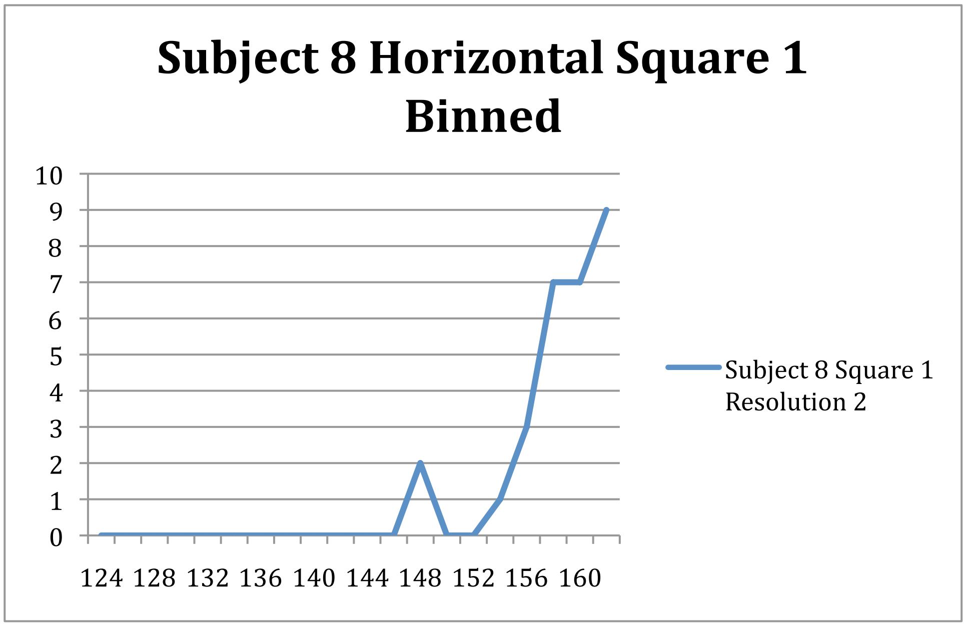

Figure 19. Subject 8 horizontal presentation square 1 results binned data; very clear transition from all "yes" answers to mostly "no" answers.

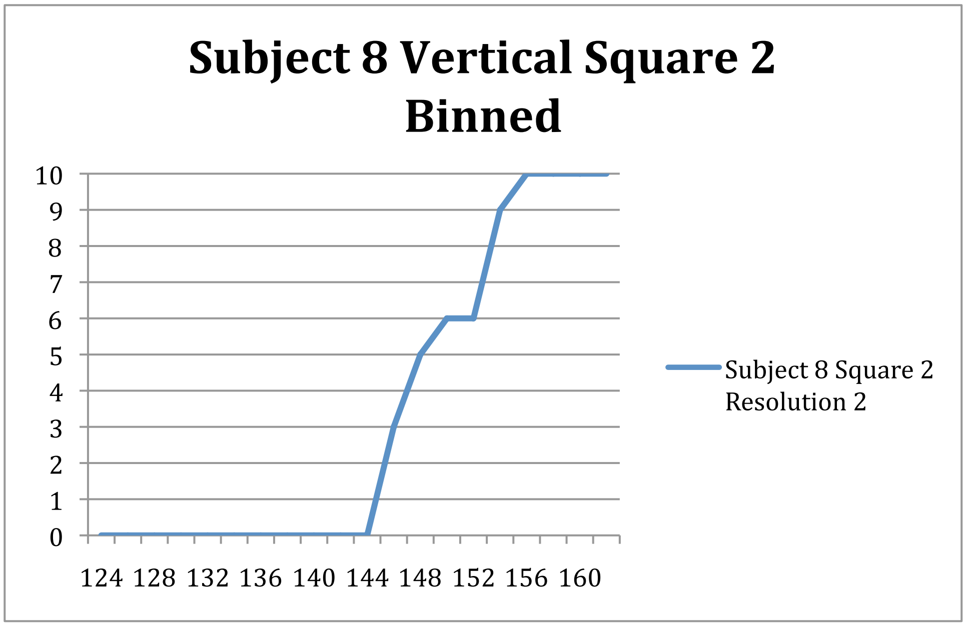

Figure 20. Subject 8 vertical presentation square 2 results binned data; very clear transition from all "yes" answers to mostly "no" answers.

Discussion

Finding systematic responding in any of the experimental conditions was challenging. For some of the observers, the task at hand was difficult regardless of the target shape.

Within the data, the psychometric curves were created so that it was possible to use all the data from a participant to draw the curve in order to see the pattern of responding over the entire stimulus range. Psychometric curves are a standard way to present the data that was collected using the method of constant stimuli. Key numbers were obtained from the individuals' psychometric curves. For many observers it was common for conditions to be unanalyzable, and therefore the data were binned. After binning, smoother psychometric curves were drawn and characteristic numbers were determined from the graphs.

All but four of the judgments for all conditions, a total of 40, were overestimated.

Comparing the individuals' data from the square presentations to the Golden Rectangle presentations with respect to the orientation did not result in any conclusive findings. The individual data for Subject 4 and Subject 8 were the only ones that could be compared.

Subject 1 had similar values for both the horizontal and vertical Golden Rectangles, but this pattern did not hold for the horizontal and vertical squares (see Table 1).

It was discovered that the scale of two pixels was too difficult to discern one shape from another, but it was not impossible because Subject 2 and Subject 8 were able to complete the tasks relatively well. Some people showed vary systematic results across all the different sized rectangles, such as Subject 8, and some did not, such as Subject 4.

Subject 4 overestimated the judgment for the horizontal Golden Rectangle by 26 pixels (see Figure 17). The horizontal square condition was unable to be scored due to the inconsistency of the judgments. Subject 4 was very unsystematic and highly variable, showing that he or she was not able to correctly judge a square or a Golden Rectangle.

Subject 7 was responding at the change level over exceptionally wide ranges of sizes. Essentially, this subject was not doing the task asked. Many of the scores could not be calculated because of the vastly unsystematic judgments. Subject 7 had the highest standard deviations for the horizontal Golden Rectangle, vertical Golden Rectangle, and the vertical square judgments, showing Subject 7's inconsistent results (see Table 1). The participant's vertical square judgments prove the inability to recognize a square, thus discrediting the vertical Golden Rectangle scores. The vertical square presentation shows that there is little relation between the increasing size of the rectangles, from smallest to largest, and the responses of the participant.

From this data, it looks like the participant did not look at the presented rectangles when judging them, but arbitrarily pressed the “Y” or “N” key.

At one extreme, at least one subject showed smooth psychometric curves at the original resolution of two pixels. Subject 8 had the lowest standard deviations in all of the experimental conditions, showing that they were very systematic in his or her judgments (see Table 1). This person seems to have been be affected by the vertical-horizontal illusion because of the overestimation of the judgments in all of the experimental conditions. Even though Subject 8 overestimated the judgments of the rectangles, their process was the most systematic of the subjects.

There was no correlation found between the mean values and standard deviations. A correlation between the y = .5 values and the standard deviations was examined and none was found, although there were hints of negative correlations. This showed that the higher the value of y = .5, the lower the variability. Because of the small sample size, the one anomalous value was skewed due to one subject’s judgments, Subject 3. R^2 is an index of association, and was used to interpret the effect size. The R^2 was found to be .35, indicating a moderately negative relationship (see Figure 13).

The individual’s x-values where the graph’s y-values equaled .5 for both the horizontal and vertical Golden Rectangles were graphed. The individual subject numbers were plotted along the x-axis, while each subject’s x-values where the graph’s y-values equaled .5 were plotted along the y-axis. The horizontal Golden Rectangle 1 judgments were greater than the vertical Golden Rectangle 2 for six participants (see Figure 14). The vertical Golden Rectangle 2 judgments were greater than the horizontal Golden Rectangle 1 for three participants (see Figure 14).

The individual's x-values where the graph's y-values equaled .5 for both the horizontal and vertical squares were graphed. The individual subject numbers were plotted along the x-axis, while each subject's x-values where the graph's y-values equaled .5 were plotted along the y-axis. The horizontal square 1 judgments were greater than the vertical square 2 for five participants (see Figure 15). The vertical square 2 judgments were greater than the horizontal square 1 judgments for three participants (see Figure 15). The patterns are similar, but the judgments do not correspond to the same participants.

Implications

The current study suggests that not all humans are able to discern a Golden Rectangle from a comparison rectangle. This implies that although it is possible to prefer the Golden Rectangle, it is not innately recognizable.

Limitations

Methodology

Data was collected from students using a computer program created using a Psychtoolbox program adapted from MullerLyer (2008) model experiment in Matlab. Most past research on preferences for shapes used paper cut outs or drawn shapes on paper. William Gaver (1996) explains the different affordances for the display of information between paper and electronic documents. Gaver (1996) explains, "paper has a resolution that is far higher compared to computers, allowing far greater subtlety and expression in the marks it displays than can be achieved on screens… paper conveys information by gradations of reflected light, not emitted light, allowing paper to merge with its surroundings more effectively than computer displays do with theirs (p. 115).” The use of computerized presentations of the shapes may have different results than drawn shapes or paper cut out shapes. The resolution of the computer used in the experiment could have affected the observers’ judgments. If reporting errors occurred due to the presentations being on the computer screen, then paper presentations may have been better.

Program Design

It was proven to be too difficult to discern the target shapes from the comparisons with a pixel difference of two.

Future Research

For future research, it would be worth it to first test the best range of pixel differences so that the task of judging would be easier for most observers to complete. It would be interesting to vary the rectangles by three, four, five, and six pixels to see where the smallest difference could be used.

It would also be interesting to see if subjects could study other rectangles, such as root three rectangles or root five rectangles, and then be able to recognize the respective proportion rectangle from comparison rectangles. This may help to decipher how special the Golden Rectangle is, or if the human mind is also drawn to proportions of other notable rectangles.

Another possibility would be to include a procedure that involved some training on the Golden Rectangle, but also on some other proportion to find out if training on the Golden Rectangle would lead to better expertise than other rectangles, or if training on any rectangle could lead to comparable improvement.

Conclusions

McCulloch (1923) did not make his recognition claim about randomly selected participants, but we began there as the strongest possible test. Because McCulloch's (1965) quotes referred to "repeated settings" and "coming to prefer" the Golden Rectangle, he was alluding to participants who were trained to prefer the Golden Rectangle. This study failed to support McCulloch's (1974) claim that humans can recognize a Golden Rectangle with a just noticeable difference of two percent. McCulloch was not incorrect with his statement, but the method used did not hold for the strongest test in determining participant's abilities. It is difficult to discern a Golden Rectangle from comparison rectangles, but it has been shown that it is possible. Some subjects proved to be very good at judging Golden Rectangles with little training, but the majority of observers performed poorly.

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Appendix A: Participant Information

Subject Number	Sex	Art Classes Taken At Trinity College	Art History Classes Taken at Trinity College
1	M	1	-
2	F	-	-
3	F	-	1
4	F	-	1
5	F	-	-
6	F	-	-
7	M	-	2
8	F	-	2+
9	F	-	2
10	F	-	1