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Bitcoin as a Global Currency: Exploring the Wild West of Cryptocurrency

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Abstract

Bitcoin, and its contemporary substitute cryptocurrencies, are an exciting new evolution in our concept of money. However, there are currently factors holding back Bitcoin, the largest player in the cryptocurrency market, from a wider mainstream acceptance and adoption. The greatest force working against cryptocurrency's ability to be an accepted method of exchange is its extreme price volatility which cannot be completely attributed to insufficient liquidity (Dyhrberg 2018). This research reexamines several GARCH models using a larger window with more observations than previous researchers, and determine that a GARCH(1,1) with an AR(6) term in the mean equation provide the best fit. After identifying the proper tool, a basket of explanatory macroeconomic variables was tested and further improved the fit. Notably, a strong relationship exists between currencies, commodities, and Bitcoin price variance furthering the common interpretation that Bitcoin exists somewhere in the ether of the two classes. Bitcoin also exhibited significant volatility responses to geopolitical events that imply a use by nefarious state actors. The objective of this project is to gain an understanding of the nature of cryptocurrency and its utilization in the macroeconomy.

Keywords: Bitcoin, cryptocurrency, money, volatility, currency, uncertainty, GARCH

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Introduction

Technologies, and society, change similarly to how water ebbs and flows. However, when these flows take the form of a torrent, their reception is often met with the same fear and uncertainty. This describes the sentiment when Bitcoin, a cryptocurrency created anonymously in 2009, flashed into the public eye in 2017. The formerly niche and shaded technology, frequently used for anonymous illicit purchases over the internet, became somewhat of a cultural phenomenon as the mainstream scrambled to explain and embrace the new age. However, this left the financial world particularly skeptical and confused, as there has been rarely such a shift in a fundamental economic pillars like currency. Which begs the question: fad or forever? Coin or crud? Why?

This research aimed to determine which GARCH model best fits Bitcoin volatility, and which macroeconomic and geopolitical explanatory variables generate the most significant responses for its price variance. A simple GARCH(1,1) with an AR(6) term in the mean equation provides the best fit for the logarithmic returns of daily Bitcoin price data out of the options tested, and variables that depend on gold and oil as well as geopolitical events involving significant nefarious actors were among the significant driving coefficients in Bitcoin price volatility.

The question of what money really is has been explored for centuries, but the answer seems to err on the side of philosophy over economics. *How is Bitcoin Money? (2016)* affirms Bitcoins status as currency and compares its underlying value system to a gold standard. However, in *Money and the Mechanism of Exchange (1875)* William Stanley Jevons defined money by four functions; a medium of exchange, a common measure of value, a standard of value, and a store of value. The predominant issue with Bitcoin at this stage is its intense volatility, the likes of which is seldom seen in other currencies or financial assets. This issue conflicts with Jevons's definition of money which seems to be crucially defined by consistency and stability. Therefore, Bitcoin's future as a viable currency is cast into doubt. The ideological purpose (and the underlying code itself) require that the cryptocurrency cannot be easily regulated by a centralized authority. But will the economy accept a mode of payment for goods and services that may expose them to a 30% overpayment by the weeks end? While the decentralized, laissez faire system that empowers the people over the fractional reserve system and central government may sound enticing to some in theory, the price we pay for stability may be far more valuable than the alternative. It is in Bitcoin holders best economic interest for the long term to facilitate such stability. Therefore, the only way forward for Bitcoin is to develop in a progressive way to prevent such destructive and rapid abuses by speculation and fraud which have tainted Bitcoins history so far. Measures have already been taken around the global economy to implement greater order and control to the blockchain's Wild West. For better or worse, the technology is here to stay and must necessarily evolve, but in which direction? Using a variety of GARCH tools, the volatility of this currency can be observed and forecasted during different periods of stability and volatility. From this perspective, perhaps a plan for a more realistic world cryptocurrency may result.

A Brief History of Bitcoin

Bitcoin was released as an open source software by an anonymous engineer (or group of engineers) under the pseudonym Satoshi Nakamoto in the wake of the 2008 financial crisis. Bitcoin is generally considered the first decentralized cryptocurrency, meaning that no institution or central bank is responsible for regulating the currency. Instead, its price is entirely at the mercy of the open market, and its supply is produced by "mining". This Bitcoin mining process draws several similarities to gold, facilitating the comparisons that have occasionally branded Bitcoin as the new gold standard. They are similar in that both have a finite quantity available, and are both enormously, and increasingly, resource intensive to acquire. However, the obvious glaring difference is that one exists as a tangible asset, and the other exists in the ether of the virtual domain. The mining process serves a fascinating and essential dual purpose in that the miner is rewarded with coins for processing transactions and adding consistent, complete, and unalterable blocks to the chain of transactions that traces back to the "Genesis block" mined by Satoshi Nakamoto in 2009. This complete ledger of transactions, famously known as a Blockchain, helps solve a double-spending problem that had doomed earlier attempts at cryptocurrencies, and offers an increased degree of insurance and security for the value of the coin.

Inscribed on the aforementioned genesis block was a brief text note, "The Times 03/Jan/2009 Chancellor on brink of second bailout for banks." This is often interpreted as a critique of the fractional reserve system, bolstering the perception that Bitcoins purpose at some level must be to subvert the power of government and big banks over the monetary system. Its use in practice is consistent with this theory. Before Bitcoin exploded into the public eye with its meteoric rise in value in 2017, it existed under the radar serving a particularly devious niche as a mode of exchange for the internet black markets. The largest of these market places was the infamous Silk Road, a dark web platform for the transaction of illicit goods and services ranging from drugs, weapons, child pornography, and even assassinations[i]. During its 30 months of service, beginning in 2011, the Silk Road marketplace exclusively accepted Bitcoin for payments at a volume of 9.9 million coins worth around $214 million[ii]. This pairing between the Silk Road and Bitcoin, while perhaps unfortunate for its long term legacy, is an obvious one. Beyond its uniquely decentralized organization, the anonymity provided by Bitcoin makes up a large part of its widespread charm. Bitcoin is "pseudonymous", meaning that while transactions can be traced back to particular online "wallets" (and in fact is easily traceable in this way because of the blockchain's public ledger mechanism), the wallet IDs do not need to be connected to real world entities, and can even be newly generated with each transaction. So, while not perfectly untraceable like cash transactions, the degree of privacy offered by Bitcoin make it perfect for online transactions, particularly of an illegal nature. This almost exclusive black market demand for Bitcoins inflated the price from $.30 per Bitcoin at the beginning of the year in 2011, to $31.50 by June, and $5.27 by years end. The FBI's closure and seizure of the Silk Road market sank the price of Bitcoin from $132.05 on October 2, 2013 to $114.45 the following day (a 13% decline).

The unfortunate history of Bitcoin continues with the story of the Mt. Gox exchange. Launched in July of 2010, by late 2013 the exchange had become the largest Bitcoin intermediary handling 70% of all Bitcoin transactions worldwide. On February 7th, 2014, Mt. Gox halted all Bitcoin withdrawals due to a bug that allowed the possibility of someone to make it seem like a transaction in their wallet did not occur, when it actually did. In quick succession, the house of cards fell, and on February 24th Mt. Gox suspended all trading and disabled the website to a blank page. A leaked crisis management document claimed the company was insolvent after having lost 744,408 Bitcoin in a theft that went undetected for years. By March, the value of Bitcoin had declined 36% as a result of the Mt. Gox controversies[iii].

This example is one of a series of Bitcoin and crypto related scams and cons that would tarnish the reputation of the emergent technology and generate deserved skepticism. It is widely recognized how Bitcoin's decentralized and pseudonymous structure lends itself to money laundering and criminal activity at large, as well as price and market manipulation within the cryptocurrency itself. Although they have been crucial to Bitcoins rise, these issues do not lend themselves to long-term security and safety for Bitcoin's growth. While the technology underlying Bitcoin is in its own right novel, and potentially revolutionary, its track record does not reflect that prospect in practice. Perhaps even now we are too early in the development of the coin for it to outgrow these early issues and mature into a stable and useful asset to facilitate international transactions and online payments.

How much do we really know about Bitcoin?

In recent years there has been a scramble to understand and classify what Bitcoin represents in the world of finance. The volatile and uncertain nature of the asset begs the question: “Which model best fits the unique characteristics of Bitcoin?”.

In 2017 Bitcoin experienced a significant asset bubble, appreciating over 100% in December alone. This was due to perhaps the general hype surrounding the mysterious new technology, or due to some long term nature inherent to the cryptocurrency asset class. Cheah and Fry (2015) determined that a bubble component contained within Bitcoin prices is substantial by fitting a general asset bubble model to the cryptocurrency. This research also makes the claim that the fundamental value of Bitcoin is zero, implying that it is almost entirely bubble driven. This conclusion cast serious doubt on the future of Bitcoin as a long term store of value. However, this result is confusing because the majority of currencies today derive their value by fiat. The counter argument is that Bitcoin is bolstered in value by the strength of the underlying Blockchain technology and is in a finite amount with many characteristics parallel to those of gold (Bjerg 2015).

In econometrics the ARCH/GARCH models (and a wide family of alternative specifications) have been among the favorites for the purpose of modeling time-varying volatility since its invention by Robert Engle in 1982. With regards to Bitcoin, researchers have sought to discover which model best fits the asset in an attempt to better understand the dynamics that govern the cryptocurrency and how those dynamics are comparable to other assets. For data from April 1, 2013 to March 21, 2016 an EGARCH model was the best fit for forecasting the Bitcoin/USD exchange rate prices (Naimy and Hayek 2018). However, earlier research by

Kisinbay (2010) stated that EGARCH performs poorly when forecasting the volatility of currencies. This conflict offers an interesting clue as to the true nature of the cryptocurrency. However, any findings currently published with regards to Bitcoin must be taken with a grain of salt, as Naimy and Hayek state themselves, because the analysis may have occurred too early in the asset's development to be indicative of its true nature in maturity. This question was faced by Stanislaw Drozdz, Robert Gebarowski, and Ludovico Minati who made the claim that Bitcoin has recently (June 2018) developed into a mature market, as it exhibits "most important complexity characteristics, related to the return distribution, temporal correlations, and multi-scaling effects, even including their generalization to discrete scale invariance". The authors also predict that the smaller cryptocurrencies (such as Ethereum and Litecoin) will follow in the larger Bitcoin's wake toward a matured market. Despite this, Bitcoin does not seem to act consistently with currencies of even developing countries, exhibiting considerably higher volatility (Kasper 2017). Paraskevi Katsiampa concluded that the most optimal model in terms of goodness-of-fit was AR(1)-CGARCH in 2017. Jeffery Chu found that for cryptocurrency data between June 22, 2014 and May 17, 2017 the IGARCH and TGARCH models provided the best fit for volatility. From a slightly different perspective Ardia, Bluteau, and Ruede 2018 looked at Bitcoin as an asset that may be responsive to regimes changes, and find that Markov-Switching GARCH models outperform single regime GARCH. Regime switching models are a clever tool to match the tendency of financial markets to change their behavior abruptly (in response to periods of regulation, changes in policy, and other secular changes) and allow the phenomena to persist for several periods (Ang and Timmermann 2012). Using a Markov Switching model could assist in determining which policy regulations, and larger economic phenomena contribute to a more stable Bitcoin price.

With all of this being said, there appears to be little clear consensus on which model best accommodates Bitcoin, which is a possible reflection of the changing dynamics of Bitcoin over time, as well as the sensitivities of the different GARCH models.

Questions regarding which assets cryptocurrencies are most similar to are also very common. In 2016, Anne Dyhrberg concluded that Bitcoin is between a currency and a commodity, because it reacts to the federal funds rate like a currency but has similar GARCH behavior to gold and has a limited market size like a commodity. Dyrhberg (2018) also found strong liquidity in Bitcoin where average quoted and effective spreads for Bitcoin were lower than on major equity exchanges during US market trading hours. Although without a GARCH analysis, Elie Bouri and Rangan Goupta found that while Bitcoin is part of an alternative economy, its price formation is affected by the aggregate commodity market and gold in particular. Understanding Bitcoin's relationship with other commodities is essential to determining its nature as an asset, and as a financial tool to hedge risk in a portfolio.

Bitcoin's early history as a tool to make illicit purchases on the internet has not been fully expunged by the seizure of the Silk Road Marketplace in 2013. The quasi-anonymous nature of Bitcoin and other cryptocurrency transactions lend themselves to black markets. There is not much research regarding current black markets and their dynamics with Bitcoin, but there have been several articles and discoveries in recent years of rogue states such as North Korea and the Islamic State of Iraq and Syria making significant purchases or laundering money through Bitcoin. The black market was essential to Bitcoin's early development, and may play a permanent role in its continuing utility as a means of payment.

Methodology and Results

Data

The dataset gathered for this analysis is based primarily on historic daily Bitcoin open prices. The range of the dataset provided by Investing.com spans from 9/13/2011 to 1/25/2019. All dates are provided in the given range including weekends and holidays. The volatility models utilized necessitate stationary data, meaning that the time series must be transformed so that the mean, variance, and autocorrelation are constant over time. For the Bitcoin open prices, the process to create a stationary transformation is to take the log of the open values, and compute the difference between the current period and the last (today minus yesterday for the daily returns).

Nine dummy variables were examined for this research in order to determine their effect and significance in explaining Bitcoins price volatility. The dummy variables are turned on, taking the value of 1, for a 30 day period following their activation date. The dummy variables are:

- Gold>BTC- March 2, 2017[iv], the date Bitcoin surpassed gold in value.
- Silk Road Closure- October 2, 2013[v], the date FBI shutdown the Silk Road marketplace.
- Fall of ISIS Raqqa- October 17, 2017[vi], the date of the U.S. declaration of the capture of the ISIS capital Raqqa.
- Fall of ISIS Mosul- July 9, 2017[vii], the date when the Islamic State of Iraq and Levant was declared officially defeated in Mosul.
- Brexit Vote- June 23, 2016[viii], the date of the Brexit referendum where 51.9% of votes were cast in favor of leaving the EU.
- Resolution2371-August 5, 2017[ix], UNSC unanimous adoption of sanctions that banned the purchase of North Korean coal, Iron, lead, and seafood.
- Resolution2375-September 11, 2017[x], UNSC unanimous adoption of new sanctions against North Korea. The resolution established a quota for selling oil to North Korea, equating to an estimated 30% reduction from previous levels. The resolution also asks all countries to inspect ships going in and out of North Korean ports.
- Resolution2397-December 22, 2017[xi], UNSC unanimous adoption of further sanctions that limited North Korean imports of refined petroleum to 500,000 barrels, and further restricted

North Korean exports. UN members were authorized to seize and impound any vessels found to illicitly supply petroleum to North Korean.

- Mt GOX closure-February 7, 2014[xii], the date Tokyo based Bitcoin exchange Mt. Gox halted withdrawals of Bitcoins.

Other quantitative variables of interest for this research include:

- 10 year Treasury Note rate-Provided by the St. Louis Federal Reserve. This data does not include weekends, so an equation was used to average the previous two days values and extrapolate the weekend rates to make the data continuous.
- Effective Federal Funds rate- Provided by the St. Louis Federal Reserve. This data is daily and continuous.
- Venezuelan Bolivar USD exchange rate- Provided by the St. Louis Federal Reserve. This data does not include weekends and certain other days, and had to be extrapolated using the previously described method.
- Natural Log Bolivar USD-The natural log of the Bolivar-USD exchange rate.
- Brent EU-The price of Brent Crude oil priced in dollars, provided by Investing.com. Data extrapolated for weekends.
- Natural Log Brent EU-The natural log of the Brent crude prices.
- Yuan USD-Chinese Yuan to USD exchange rate, provided by the St. Louis Federal Reserve. Data extrapolated for weekends.
- Natural log Yuan USD-The natural log of the Yuan-USD exchange rate.
- USDRUB- One US Dollar to Russian Ruble exchange rate open values, provided by Business Insider. Daily and continuous.
- USDindex- a measure of the value of the U.S. dollar relative to the value of a basket of currencies of the majority of the U.S.'s most significant trading partners. Daily continuous data provided by Business Insider.
- USEURO- USD to Euro foreign exchange rate, US dollars to one Euro. Daily, extrapolated for weekends, and not seasonally adjusted. Provided by the St. Louis Federal Reserve.
- USDPound- USD to Pound. US dollars to one British Pound. Daily, extrapolated for weekends, and not seasonally adjusted. Provided by the St. Louis Federal Reserve.
- ETHUSD-Ethereum, the second largest cryptocurrency by market cap, exchange rate to USD. Data is daily and continuous from August 7th, 2015 to January 25th, 2019.
- Gold price-The price of gold per troy ounce, provided by the St. Louis Federal Reserve.Gold Fixing Price 10:30 A.M. (London time) in London Bullion Market, based on U.S. Dollars per Troy Ounce, Daily, Not Seasonally Adjusted.
- Difficulty- A metric calculated by Blockchain.com to show how difficult it is to find a new block. The difficulty is adjusted periodically as a function of how much hashing power has been deployed by the network miners
- Trasacntions24hours-The number of daily confirmed Bitcoin transactions provided by Blockchain.com

Models

<u>ARCH</u>

Autoregressive Conditional Heteroskedasticity models, referred to as ARCH models, have enabled econometricians to model the attitude of investors toward expected returns as well as risk and uncertainty. The assumption that the variance is constant over time, a homoskedasticity assumption, does not hold up in practice where instead financial time series data tends to exhibit volatility clustering (periods of unusually high volatility, followed by periods of relative calm). Since it is expected that the conditional variance for Bitcoin is not constant, it is beneficial to model simultaneously the mean and the variance of the series (Engle 1982). The mean equation in this case would take the form:

$$Y_t = a + \beta' X_t + u_t$$

Normally, the residuals (u_t), are assumed to be independently distributed with a zero mean and constant variance of σ^2. However, to allow the variance of the residuals to depend on history, and therefore have heteroskedasticity, Engle proposed to have the variance depend on the one lagged period of the squared error term:

$$\sigma_t^2 = \gamma_0 + \gamma_1 u_{t-1}^2$$

The combination of these models represents the ARCH(1) process. The (1) means that the model takes the squared residual from the last period (u_{t-1}^2), but can be extended to accommodate additional lagged periods (q) as an ARCH(q). Typically, the conditional variance equation in volatility models assumes the form h_t instead of σ_t^2, and will be as such for the remainder of this section.

In order to determine if the ARCH model is appropriate for this Bitcoin data set, we must first test for the presence of the necessary ARCH effects. Based on Figure 1, it can be seen clearly that there are periods of greater and lesser volatility, evidence of volatility clustering or pooling, implying a good likelihood of heteroskedastic ARCH effects in the residuals of this model.

The first step to confirming this hypothesis is to use an Ordinary Least Squares regression of the logged returns, and then check for such effects using a Breusch-Pagan and Breusch-Godfry test. The output for the Ordinary Least Squares analysis is printed on Table 1, and the tests for serial correlation using the Breusch-Godfry as well as the Breusch-Pagan test for heteroskedasticity are seen in Table 2.

The Breusch-Godfry test for serial correlation shows the null hypothesis of no serial correlation at up to one lag cannot be rejected at the 5%, and the Durbin Watson value near 2 also implies no serial correlation.

The Obs*R-Squared (or $T * R^2$ statistic) in the Breusch-Pagan test is 314.6878 and has a probability value of .0000, which suggests that we reject the null hypothesis of homoskedasticity. Testing for higher order ARCH effects, illustrated in Table 4 with 6 lags, produces an even higher Obs*R-squared value of 352.6369 and is also highly significant. The lagged squared residuals in this example are mostly statistically significant but varying at different lags. Based on this testing, it is clear that an ARCH model will provide better results.

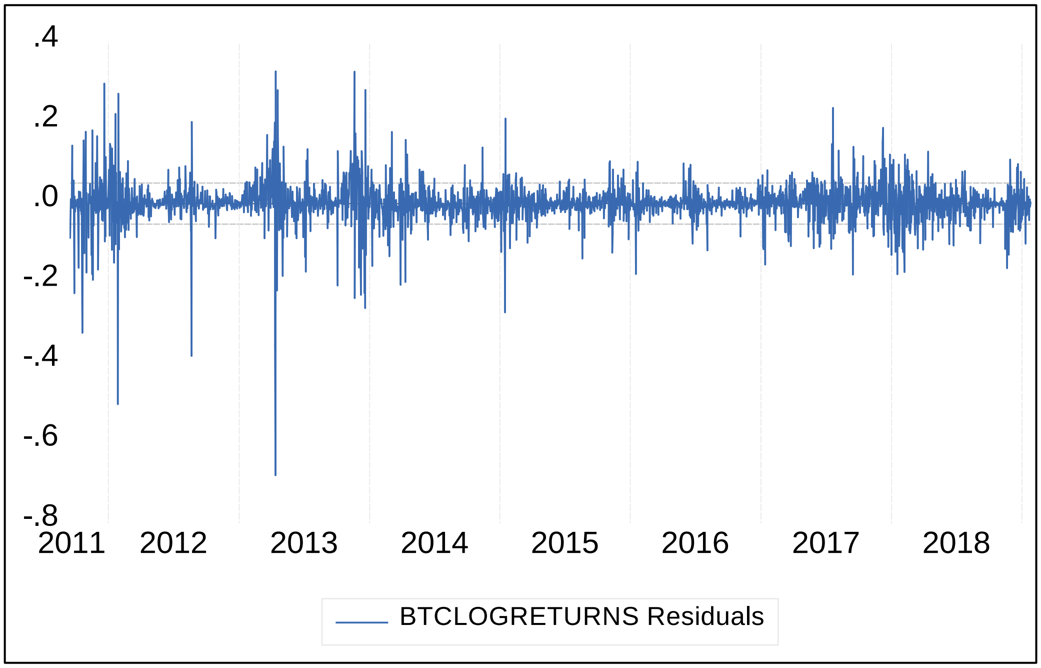

Figure 1 Graph of the residuals for the logged returns of Bitcoin exhibiting clear volatility pooling.

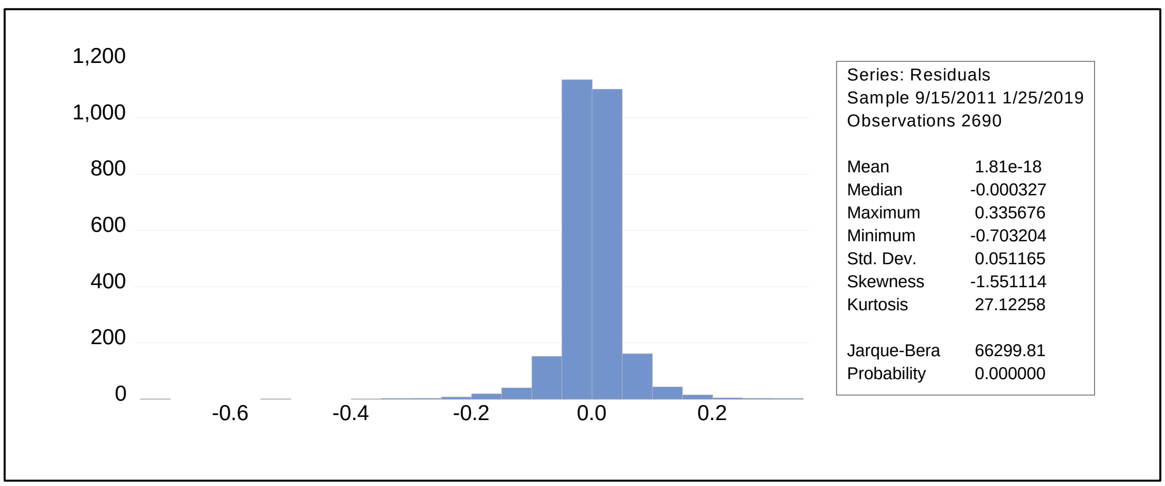

Figure 2 Histogram normality test for the Residuals. The residuals are not normally distributed but exhibit clear symmetry and a mean of zero.

Dependent Variable: BTCLOGRETURNS
Method: Least Squares
Date: 03/19/19 Time: 17:21
Sample (adjusted): 9/15/2011 1/25/2019
Included observations: 2690 after adjustments

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	0.002456	0.000988	2.486558	0.0130
BTCLOGRETURNS(-1)	-0.022690	0.019281	-1.176826	0.2394

R-squared	0.000515	Mean dependent var	0.002402
Adjusted R-squared	0.000143	S.D. dependent var	0.051178
S.E. of regression	0.051175	Akaike info criterion	-3.106406
Sum squared resid	7.039426	Schwarz criterion	-3.102021
Log likelihood	4180.116	Hannan-Quinn criter.	-3.104820
F-statistic	1.384920	Durbin-Watson stat	2.000724
Prob(F-statistic)	0.239369		

Table 1 The least squares regression of the log returns of Bitcoin open price.

Breusch-Godfrey Serial Correlation LM Test:
Null hypothesis: No serial correlation at up to 1 lag

F-statistic	3.084352	Prob. F(1,2687)	0.0792
Obs*R-squared	3.084256	Prob. Chi-Square(1)	0.0791

Test Equation:
Dependent Variable: RESID
Method: Least Squares
Date: 03/19/19 Time: 18:16
Sample: 9/15/2011 1/25/2019
Included observations: 2690
Presample missing value lagged residuals set to zero.

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	-0.002940	0.001944	-1.512776	0.1305
BTCLOGRETURNS(-1)	1.232061	0.701801	1.755570	0.0793
RESID(-1)	-1.232991	0.702066	-1.756232	0.0792

R-squared	0.001147	Mean dependent var	1.81E-18
Adjusted R-squared	0.000403	S.D. dependent var	0.051165
S.E. of regression	0.051155	Akaike info criterion	-3.106810
Sum squared resid	7.031355	Schwarz criterion	-3.100233
Log likelihood	4181.659	Hannan-Quinn criter.	-3.104431
F-statistic	1.542176	Durbin-Watson stat	2.002020
Prob(F-statistic)	0.214104		

Heteroskedasticity Test: ARCH

F-statistic	356.1310	Prob. F(1,2687)	0.0000
Obs*R-squared	314.6878	Prob. Chi-Square(1)	0.0000

Test Equation:
Dependent Variable: RESID^2
Method: Least Squares
Date: 04/02/19 Time: 09:44
Sample (adjusted): 9/16/2011 1/25/2019
Included observations: 2689 after adjustments

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	0.001719	0.000247	6.958339	0.0000
RESID^2(-1)	0.342085	0.018127	18.87143	0.0000

R-squared	0.117028	Mean dependent var	0.002615
Adjusted R-squared	0.116699	S.D. dependent var	0.013380
S.E. of regression	0.012575	Akaike info criterion	-5.913525
Sum squared resid	0.424873	Schwarz criterion	-5.909139
Log likelihood	7952.734	Hannan-Quinn criter.	-5.911938
F-statistic	356.1310	Durbin-Watson stat	2.034471
Prob(F-statistic)	0.000000		

Table 2 Top: Breusch-Godfry Test with 1 lag, ARCH(1). Bottom: Breusch-Pagan ARCH test for heteroskedasticity.

Table 6 below shows the results of the ARCH(1) model with our logarithmic returns for the Bitcoin data. It took the model 10 iterations to achieve convergence. The autoregressive term in the mean equation, with a value of -.0209 is not statistically significant, but both terms in the variance equation are highly statistically significant. Making an adjustment to the mean equation by incorporating an autoregressive term for the last two periods (ar(2)), greatly improves the significance of the mean equation, as well and improves the fit for subsequent models. The ARCH(1) model takes the form:

$$Y_t = .00192 - .021Y_{t-1} + u_t$$

(2.34) (-1.47)

$$u_t|\Omega_t \sim iid\ N(0, h_t)$$
$$h_t = .001541 + .38237u_{t-1}^2$$

(96.74) (19.8)

A quick test of an ARCH(6) setup, as shown in Table 7, presents preferable results. For ARCH models, the short term variance of the series is a function of the immediate past values of the squared error term. The ARCH(6) extends this period to around a week (6 days), which would appear to be a more reasonable when considering the nature and variability of the Bitcoin series. Our mean equation has now become highly statistically significant as specified with the AR(1) term. Further, all of the γ's (the coefficients on the lagged squared residuals) are positive and statistically significant as desired and this model gives us a significantly better Schwarz criterion value. The Schwarz criterion, which is closely related to the Akaike Information Criterion, is based on the likelihood function and balances added parameters with the risk of overfitting. Typically, the model with the smallest or most negative Schwarz information criterion is preferred. This leads us to the implementation of a GARCH model to solve this dilemma.

Heteroskedasticity Test: ARCH

F-statistic	67.48619	Prob. F(6,2677)	0.0000
Obs*R-squared	352.6369	Prob. Chi-Square(6)	0.0000

Test Equation:
Dependent Variable: RESID^2
Method: Least Squares
Date: 04/02/19 Time: 14:06
Sample (adjusted): 9/21/2011 1/25/2019
Included observations: 2684 after adjustments

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	0.001365	0.000255	5.351529	0.0000
RESID^2(-1)	0.323847	0.019261	16.81390	0.0000
RESID^2(-2)	0.037459	0.020246	1.850192	0.0644
RESID^2(-3)	-0.018707	0.020201	-0.926014	0.3545
RESID^2(-4)	0.079008	0.020201	3.911084	0.0001
RESID^2(-5)	-0.026285	0.020246	-1.298286	0.1943
RESID^2(-6)	0.083079	0.019260	4.313573	0.0000

R-squared	0.131385	Mean dependent var	0.002618
Adjusted R-squared	0.129438	S.D. dependent var	0.013392
S.E. of regression	0.012495	Akaike info criterion	-5.924390
Sum squared resid	0.417939	Schwarz criterion	-5.909015
Log likelihood	7957.531	Hannan-Quinn criter.	-5.918828
F-statistic	67.48619	Durbin-Watson stat	2.005038
Prob(F-statistic)	0.000000		

Table 4 Breusch-Pagan Test with 6 lag, or ARCH(6).

Dependent Variable: BTCLOGRETURNS
Method: ML ARCH - Normal distribution (BFGS / Marquardt steps)
Date: 03/24/19 Time: 14:37
Sample (adjusted): 9/15/2011 1/25/2019
Included observations: 2690 after adjustments
Convergence achieved after 10 iterations
Coefficient covariance computed using outer product of gradients
Presample variance: backcast (parameter = 0.7)
GARCH = C(3) + C(4)*RESID(-1)^2

Variable	Coefficient	Std. Error	z-Statistic	Prob.
C	0.001918	0.000820	2.340185	0.0193
BTCLOGRETURNS(-1)	-0.020929	0.014161	-1.477911	0.1394
		Variance Equation		
C	0.001541	1.59E-05	96.73761	0.0000
RESID(-1)^2	0.382370	0.019310	19.80180	0.0000

R-squared	0.000403	Mean dependent var	0.002402
Adjusted R-squared	0.000031	S.D. dependent var	0.051178
S.E. of regression	0.051177	Akaike info criterion	-3.367029
Sum squared resid	7.040215	Schwarz criterion	-3.358260
Log likelihood	4532.654	Hannan-Quinn criter.	-3.363858
Durbin-Watson stat	2.003966		

Table 5 ARCH(1) model for log returns of Bitcoin.

Dependent Variable: BTCLOGRETURNS
Method: ML ARCH - Normal distribution (BFGS / Marquardt steps)
Date: 03/24/19 Time: 15:29
Sample (adjusted): 9/15/2011 1/25/2019
Included observations: 2690 after adjustments
Convergence achieved after 27 iterations
Coefficient covariance computed using outer product of gradients
Presample variance: backcast (parameter = 0.7)
GARCH = C(3) + C(4)*RESID(-1)^2 + C(5)*RESID(-2)^2 + C(6)*RESID(-3)^2
+ C(7)*RESID(-4)^2 + C(8)*RESID(-5)^2 + C(9)*RESID(-6)^2

Variable	Coefficient	Std. Error	z-Statistic	Prob.
C	0.001242	0.000625	1.986273	0.0470
BTCLOGRETURNS(-1)	0.089672	0.017989	4.984965	0.0000
		Variance Equation		
C	0.000475	1.30E-05	36.53360	0.0000
RESID(-1)^2	0.278689	0.018179	15.33025	0.0000
RESID(-2)^2	0.137222	0.016036	8.557377	0.0000
RESID(-3)^2	0.094285	0.012287	7.673864	0.0000
RESID(-4)^2	0.136522	0.014859	9.187893	0.0000
RESID(-5)^2	0.232377	0.019900	11.67748	0.0000
RESID(-6)^2	0.125061	0.014267	8.765776	0.0000

R-squared	-0.012455	Mean dependent var	0.002402
Adjusted R-squared	-0.012832	S.D. dependent var	0.051178
S.E. of regression	0.051506	Akaike info criterion	-3.591506
Sum squared resid	7.130773	Schwarz criterion	-3.571775
Log likelihood	4839.576	Hannan-Quinn criter.	-3.584369
Durbin-Watson stat	2.218569		

Table 6 ARCH(6) model for log returns of Bitcoin.

GARCH

The GARCH model has slightly more sophistication than the ARCH setup. In a GARCH model, the lagged conditional variance terms are included as autoregressive terms. The GARCH(p,q) has the following form:

$$Y_t = a + \beta' X_t + u_t$$
$$u_t | \Omega_t \sim iid\ N(0, h_t)$$
$$h_t = \gamma_0 + \sum_{i=1}^{p} \delta_i\, h_{t-i} + \sum_{j=i}^{q} \gamma_j\, u_{t-j}^2$$

The mean equation is the same as the previous ARCH model. The key difference comes in the variance scaling parameter h_t, which now depends on the both the past values of shocks (the past values of the residuals squared, shown by u_{t-j}^2) and its own past values (h_{t-i}). If p = 0 the model is reduced to the ARCH(q). Further, the implementation of the GARCH model is appropriate and useful in this example, because the GARCH (1,1) is an equivalent alternative to an infinite ARCH(q) process with coefficients that decline geometrically, thereby capturing historical information with fewer added parameters and fewer degrees of freedom lost. The proof for this relationship can be found in the appendix.

The benefits of this process are illustrated in Table 7, where the results of the GARCH (1,1) for the Bitcoin returns produce an even smaller Schwarz criterion, and all positive and statistically significant coefficients for our variance equation. Our model now takes the following form:

$$Y_t = .0015 + .0407 Y_{t-1} + \hat{u}_t$$
(2.14) (2.01)
$$h_t = .0000499 + .8339 h_{t-1} + .1657 \hat{u}_{t-1}^2$$
(18.81) (146.95) (23.96)

Dependent Variable: BTCLOGRETURNS
Method: ML ARCH - Normal distribution (BFGS / Marquardt steps)
Date: 03/26/19 Time: 09:09
Sample (adjusted): 9/15/2011 1/25/2019
Included observations: 2690 after adjustments
Convergence achieved after 22 iterations
Coefficient covariance computed using outer product of gradients
Presample variance: backcast (parameter = 0.7)
GARCH = C(3) + C(4)*RESID(-1)^2 + C(5)*GARCH(-1)

Variable	Coefficient	Std. Error	z-Statistic	Prob.
C	0.001502	0.000701	2.143821	0.0320
BTCLOGRETURNS(-1)	0.040739	0.020214	2.015336	0.0439
		Variance Equation		
C	4.99E-05	2.65E-06	18.81102	0.0000
RESID(-1)^2	0.165748	0.006917	23.96234	0.0000
GARCH(-1)	0.833912	0.005675	146.9550	0.0000

R-squared	-0.003755	Mean dependent var	0.002402
Adjusted R-squared	-0.004129	S.D. dependent var	0.051178
S.E. of regression	0.051284	Akaike info criterion	-3.631405
Sum squared resid	7.069500	Schwarz criterion	-3.620444
Log likelihood	4889.240	Hannan-Quinn criter.	-3.627440
Durbin-Watson stat	2.124521		

Table 7 GARCH(1,1) model for log returns of Bitcoin.

GARCH-M (GARCH in mean)

Within the family of GARCH models there exists alternative specifications that allow for a more nuanced analysis based on the volatility model. One of these is the “GARCH in mean”, or GARCH-M model. This alteration allows for the conditional mean to depend on its own conditional variance. This model is useful for securities where the return may be dependent on its volatility (risk). In other words, this model helps describe situations where investors seek a premium as compensation for holding a risky asset, which may be the case when it comes to Bitcoin. Our models now take the form:

$$Y_t = a + \beta' X_t + \theta h_t + u_t$$
$$u_t | \Omega_t \sim iid\ N(0, h_t)$$
$$h_t = \gamma_0 + \sum_{i=1}^{p} \delta_i\, h_{t-i} + \sum_{j=i}^{q} \gamma_j\, u_{t-j}^2$$

Here, the variance equation remains unchanged, but the mean equation takes on the additional conditional variance value (θh_t). Alternatively, taking the square root of the variance series utilizes the standard deviation instead, producing potentially different results. For the case of the standard deviation in the mean equation, the model would assume the following form:

$$Y_t = a + \beta' X_t + \theta\sqrt{h_t} + u_t$$

Neither the variance nor the standard deviation iterations of our GARCH-M produced a statistically significant GARCH coefficient in the mean equation, as shown below in Table 9. This suggests that for Bitcoin returns there is little feedback from the conditional variance to the conditional mean. The coefficient for the variance equation is slightly more significant than the standard deviation version, implying that if there is an effect on the risk of the mean return, it is likely better captured by the variance. Further, the Schwarz information criterion dropped slightly from the GARCH(1,1), indicating a weaker fit.

Dependent Variable: BTCLOGRETURNS
Method: ML ARCH - Normal distribution (BFGS / Marquardt steps)
Date: 03/27/19 Time: 09:17
Sample (adjusted): 9/15/2011 1/25/2019
Included observations: 2690 after adjustments
Convergence achieved after 24 iterations
Coefficient covariance computed using outer product of gradients
Presample variance: backcast (parameter = 0.7)
GARCH = C(4) + C(5)*RESID(-1)^2 + C(6)*GARCH(-1)

Variable	Coefficient	Std. Error	z-Statistic	Prob.
GARCH	0.655801	0.482430	1.359370	0.1740
C	0.000778	0.000890	0.873754	0.3823
BTCLOGRETURNS(-1)	0.040654	0.020304	2.002233	0.0453
Variance Equation				
C	5.05E-05	2.70E-06	18.68165	0.0000
RESID(-1)^2	0.167806	0.007073	23.72393	0.0000
GARCH(-1)	0.832201	0.005752	144.6711	0.0000

R-squared	-0.000708	Mean dependent var	0.002402
Adjusted R-squared	-0.001453	S.D. dependent var	0.051178
S.E. of regression	0.051215	Akaike info criterion	-3.631876
Sum squared resid	7.048037	Schwarz criterion	-3.618722
Log likelihood	4890.873	Hannan-Quinn criter.	-3.627118
Durbin-Watson stat	2.106374		

Dependent Variable: BTCLOGRETURNS
Method: ML ARCH - Normal distribution (BFGS / Marquardt steps)
Date: 03/27/19 Time: 09:21
Sample (adjusted): 9/15/2011 1/25/2019
Included observations: 2690 after adjustments
Convergence achieved after 26 iterations
Coefficient covariance computed using outer product of gradients
Presample variance: backcast (parameter = 0.7)
GARCH = C(4) + C(5)*RESID(-1)^2 + C(6)*GARCH(-1)

Variable	Coefficient	Std. Error	z-Statistic	Prob.
@SQRT(GARCH)	0.067791	0.050644	1.338587	0.1807
C	-0.000554	0.001717	-0.322446	0.7471
BTCLOGRETURNS(-1)	0.040767	0.020287	2.009474	0.0445
Variance Equation				
C	5.04E-05	2.74E-06	18.42635	0.0000
RESID(-1)^2	0.167552	0.007022	23.86258	0.0000
GARCH(-1)	0.832406	0.005695	146.1672	0.0000

R-squared	-0.001359	Mean dependent var	0.002402
Adjusted R-squared	-0.002104	S.D. dependent var	0.051178
S.E. of regression	0.051232	Akaike info criterion	-3.631501
Sum squared resid	7.052624	Schwarz criterion	-3.618347
Log likelihood	4890.369	Hannan-Quinn criter.	-3.626743
Durbin-Watson stat	2.122115		

Table 9 GARCH-M(1,1) model for the logarithmic returns of Bitcoin using the variance method (above), and the standard deviation method (below).

TGARCH (Threshold GARCH)

For an ARCH/GARCH model, large positive shocks have the same effect on the volatility of the series as large negative shocks of the same magnitude due to the residual term being squared. In reality negative shocks and positive shocks have asymmetric impacts on volatility, where it has been observed that typically "bad news" has a larger impact on the volatility than "good news". This approach seems particularly applicable to Bitcoin that has developed an infamous reputation for its proclivity toward inflationary bubbles (Fry 2018). The TGARCH finds asymmetries in positive and negative shocks by adding into the variance equation a multiplicative dummy variable to check whether there is a statistically significant difference when shocks are negative. This variance equation for TGARCH(1,1) has the form:

$$h_t = \gamma_0 + \gamma u_{t-1}^2 + \theta u_{t-1}^2 d_{t-1} + \delta h_{t-1}$$

The added dummy variable is d_t, that takes the value of 1 for u_t< 0, and 0 otherwise. Good news has the impact of γ, and bad news has the impact of $\gamma + \theta$. If $\theta = 0$ the news impact is symmetric, and if $\theta > 0$ there is asymmetry. In our case, the RESID(-1)^2*(RESID(-1)<0) coefficient is not positive, and not statistically significant for our given time sample. With these results we cannot conclude whether there is a statistically significant difference when shocks are negative or positive. This question can be revisited with the EGARCH model in the next section. The variance model in this case takes the form:

$$h_t = .000049 + .1738u_{t-1}^2 - .0189\theta u_{t-1}^2 + .8356h_{t-1}$$

(18.53) (17.92) (-1.596) (147.99)

Dependent Variable: BTCLOGRETURNS
Method: ML ARCH - Normal distribution (BFGS / Marquardt steps)
Date: 03/27/19 Time: 10:14
Sample (adjusted): 9/15/2011 1/25/2019
Included observations: 2690 after adjustments
Convergence achieved after 25 iterations
Coefficient covariance computed using outer product of gradients
Presample variance: backcast (parameter = 0.7)
GARCH = C(3) + C(4)*RESID(-1)^2 + C(5)*RESID(-1)^2*(RESID(-1)<0) + C(6)*GARCH(-1)

Variable	Coefficient	Std. Error	z-Statistic	Prob.
C	0.001611	0.000724	2.225447	0.0261
BTCLOGRETURNS(-1)	0.038109	0.020236	1.883236	0.0597
Variance Equation				
C	4.90E-05	2.64E-06	18.53352	0.0000
RESID(-1)^2	0.173833	0.009701	17.91980	0.0000
RESID(-1)^2*(RESID(-1)<0)	-0.018992	0.011897	-1.596428	0.1104
GARCH(-1)	0.835607	0.005646	147.9913	0.0000

R-squared	-0.003369	Mean dependent var	0.002402
Adjusted R-squared	-0.003743	S.D. dependent var	0.051178
S.E. of regression	0.051274	Akaike info criterion	-3.630996
Sum squared resid	7.066784	Schwarz criterion	-3.617842
Log likelihood	4889.689	Hannan-Quinn criter.	-3.626238
Durbin-Watson stat	2.119525		

Table 10 TGARCH(1,1) model for log returns of Bitcoin.

EGARCH (Exponential GARCH)

The exponential GARCH model, on the other hand, makes the leverage effect exponential rather than quadratic, and therefore the estimates of the conditional variance are guaranteed to be non-negative, and also allows for tests of asymmetries. The more complicated EGARCH model takes the form:

$$\log(h_t) = \gamma + \sum_{j=1}^{q} \zeta_j \left|\frac{u_{t-j}}{\sqrt{h_{t-j}}}\right| + \sum_{j=1}^{q} \xi_j \frac{u_{t-j}}{\sqrt{h_{t-j}}} + \sum_{i=1}^{p} \delta_i \log(h_{t-i})$$

To test for asymmetries, the ξ term is informative. When ξ_j is negative, then positive shocks (or “good” news) generate less volatility than negative shocks. In our results below, this term (C5) is slightly negative, which would indicate bad news has a larger effect on the volatility than good news, though the coefficient is statistically insignificant for this time period. Therefore, we cannot conclude based on these results any asymmetries in volatility in response to positive and negative shocks. However, the EGARCH term, C(6), becomes far more significant and impactful in this model when compared to the base GARCH(1,1) model. This would indicate that this model can better track the time varying variance of the data more effectively, making it a better forecasting tool which is consistent with V.Y. Naimy and M.R. Hayek’s conclusion in 2018.

Dependent Variable: BTCLOGRETURNS
Method: ML ARCH - Normal distribution (BFGS / Marquardt steps)
Date: 03/28/19 Time: 08:54
Sample (adjusted): 9/15/2011 1/25/2019
Included observations: 2690 after adjustments
Convergence achieved after 53 iterations
Coefficient covariance computed using outer product of gradients
Presample variance: backcast (parameter = 0.7)
LOG(GARCH) = C(3) + C(4)*ABS(RESID(-1)/@SQRT(GARCH(-1))) + C(5)
*RESID(-1)/@SQRT(GARCH(-1)) + C(6)*LOG(GARCH(-1))

Variable	Coefficient	Std. Error	z-Statistic	Prob.
C	0.001868	0.000449	4.157045	0.0000
BTCLOGRETURNS(-1)	0.057659	0.018484	3.119401	0.0018
		Variance Equation		
C(3)	-0.468815	0.018590	-25.21837	0.0000
C(4)	0.304808	0.008927	34.14355	0.0000
C(5)	-0.005313	0.006948	-0.764700	0.4445
C(6)	0.958411	0.002178	440.1150	0.0000

R-squared	-0.006003	Mean dependent var	0.002402
Adjusted R-squared	-0.006377	S.D. dependent var	0.051178
S.E. of regression	0.051341	Akaike info criterion	-3.632964
Sum squared resid	7.085330	Schwarz criterion	-3.619810
Log likelihood	4892.336	Hannan-Quinn criter.	-3.628206
Durbin-Watson stat	2.157755		

Table 11 EGARCH(1,1) results for the logarithmic returns of Bitcoin.

Explanatory Variables in the Variance Equation

GARCH models can also be manipulated to accommodate explanatory variables into the conditional variance equation. A GARCH(p,q) that includes explanatory variables would be in the form:

$$h_t = \gamma_0 + \sum_{i=1}^{p} \delta_i h_{t-j} + \sum_{j=1}^{q} \gamma_j u_{t-j}^2 + \sum_{k=1}^{m} \mu_k X_k$$

The last term X_k represents the additional explanatory variable set. Multiple explanatory variables can be added at a time. Explanatory variables were originally added in like groups to the variance equation. For these GARCH(1,1) models, an AR(6) variable was added to the mean equation which was highly significant in all of the tests, and improved the fit of the graph. The autoregressive term with six lags is a reasonable addition given that this data is daily, so the mean will be dependent on its previous week's values.

The first selection of variables to be tested are the group of currencies. The basket of currencies was selected based on their significance to the cryptocurrency market as the top five countries for total Bitcoin transaction volume. As of July 2018, the all-time Country volumes were ranked as follows: United States (22.77%), followed by Russia (16.55%), U.K.(11.67%), Venezuela(11.18%), and China (9.83%)[xiii]. The natural log of the Venezuelan Bolivar was taken because of the near 2 million percent inflation rate in recent months[xiv]. Correlations for the currencies, including the Euro and Ethereum (the second largest cryptocurrency by market cap) can be seen in the appendix. Inputting each of the five currencies into the variance equation produces the results in model one in Table 12. The Yuan and the Pound, the least correlated currencies to Bitcoin price, do not have a statistically significant effect on Bitcoin variance. The coefficient on the Dollar Ruble exchange rate implies a negative effect on the volatility of

Bitcoin. The USD-Ruble variable increases in value when the dollar appreciates, or when the Ruble depreciates since it is measured as Rubles to one Dollar. The direction of this exchange rate makes the dynamics consistent between the Ruble and USDindex. There is also a negative coefficient on the Yuan exchange rate, but the coefficients on both the Renminbi and the British pound are statistically insignificant. The removal of the Yuan from the model, exhibited in column two, improves the significance of all variables as well as the value of the Schwarz criterion. This result makes sense because the Yuan is artificially insulated from market forces by the Chinese Central Bank[xv].

The next batch of variables are the geopolitical dummy variables in column three. Here, the 30-day window after the initial Brexit vote displayed a small, but very statistically significant and positive impact on Bitcoin price variance. The other significant variable in this group is UN Resolution 2375. This resolution sanctioned and restricted the UN member's oil trade with North Korea. Unlike other heavily sanctioned economies, such as Iran, Syria, or Russia, North Korea lacks the natural resources to meet their energy needs. Since the Resolution was put into effect, the United States government and UN council have gathered hard evidence that North Korea has been skirting sanctions by way of ship-to-ship transfers[xvi]. It is unknown how many purchases have been made in total, but one UN discovery of a single open seas transfer comprised 57,623.491 barrels, worth about $5,730,886. In theory, significant crude oil purchases laundered through bitcoin would increase volatility, which explains the positive value of this coefficient. The end of the Islamic State's control of Mosul, the capital of their self-declared caliphate, was almost significant at the 5% level, with a positive coefficient of .000391. Upon further investigation, there is evidence of ISIS utilizing the Bitcoin platform to subvert western financial sanctions. For example, in 2017 a Long Island woman named Zoobia Shahnaz pleaded guilty for funding ISIS through bank fraud and Bitcoin money laundering[xvii]. Beyond that, the size and sophistication of the ISIS economy was not trivial. The group spent an estimated $10 million per month on payroll, acquired $2 billion with the capture of Mosul in 2014, and made revenues of $450 million a year in oil sales which only made up around a half of their total income[xviii].

Column four looked at the impact of the closures of the Silk Road Marketplace and the Mt.Gox exchange on Bitcoin price variance. The Mt. Gox closure gave a smaller, but positive coefficient of .000362 and was significant at the 5% level. The date that this dummy variable is set to marked the beginning of the end for the Mt. Gox exchange, where they halted Bitcoin withdrawals from their holdings after they disclosed a major security breach and theft. Because of the halt, the dampened and positive coefficient is reasonable. Also, because the collapse of the company and exchange was a prolonged process, it is possible the 30 day window does not catch some of the turbulence caused by the eventual liquidation. The FBI shutdown of the Silk Road Market place gives by far the largest coefficient of the variables tested and is highly statistically significant. The uncertainty caused by the removal of the most significant Bitcoin market should be expected to have a massive effect on price variance, especially because of the novelty of Bitcoin at the time.

The final column attempts to illustrate the responsiveness of Bitcoin price variance to a small pool of important macroeconomic rates and commodities that may be related to Bitcoin. Surprisingly, the addition of the gold price variable showed no relationship to the variance of Bitcoin, and its inclusion in the model causes the other variables in this group to become insignificant as well. Also, the dummy variable representing the date when Bitcoin surpassed gold price did not result in a statistically significant difference in volatility. The results for the model running gold value in the variance equation can be found in the appendix, as well as the correlations for this group. Because of this, gold was excluded from the model in column five of table 12 but will be revisited in a separate model with some transformations. Without it, we see a significant positive coefficient for the price of Brent crude oil and the 10 year treasury yield, and a statistically insignificant coefficient for the federal funds rate. When retested by itself, the Federal Funds rate did yield a significant and positive coefficient in the variance equation. This result implies weak robustness for this variable, and the sign of the coefficient is the opposite of the results from Dyhrberg in 2016. This standalone model can be found in the appendix. When the price of crude oil rises, the price variance of Bitcoin increases. An explanation for this relationship is consistent with the theory explaining the statistically significant effects of Resolution 2375, and for that matter the Russian Ruble, The Ruble is highly correlated to the price of crude oil, as the commodity makes up a significant portion of their exports, so the connection between these two variables could be explained partially by their connection with oil. oil may be contributing to price variance of Bitcoin by way of oil purchases laundered through Bitcoin, or because of the energy intensive process required to mine Bitcoin.

The Difficulty metric and the amount of transactions in the last 24 hours provided by Blockchain.com provided no useful results.

The relationship with gold and Ethereum to Bitcoin is better captured in their log price returns instead of their open prices. These transformations, and their effect on Bitcoin variance can be found in Table 13. The coefficient on the log rate of change of gold is positive and for Ethereum it is negative, and both are highly statistically significant. Therefore, when gold experiences greater volatility, Bitcoin also experiences greater price variance. The opposite behavior is observed for Ethereum, where increased price variance in Ethereum actually has a dampening effect on Bitcoin price variance. These results make sense if those who invest in gold also see Bitcoin as a comparable asset, and see Ethereum as a substitute asset.

Sample (adjusted): 8 1900
Included observations: 1893 after adjustments
Convergence achieved after 43 iterations

Variable	1	2	3	4	5
C	0.00175 (2.02)*	0.001711 (1.97)*	0.001428 (1.88)	0.001875 (2.75)**	0.002912 (3.71)***
AR(6)	0.099811 (3.90)****	0.100723 (3.99)****	0.085986 (4.10)****	0.09139 (4.33)****	0.090309 (4.38)****
Variance Equation					
C	5.00E-05 (0.19)	-0.000336 (-4.22)****	5.50E-05 (20.15)****	3.20E-05 (15.14)****	-0.000152 (-13.22)****
RESID(-1)^2	0.202574 (22.16)****	0.20642 (21.56)****	0.168841 (24.32)****	0.1844 (25.07)****	0.196119 (232.87)****
GARCH(-1)	0.802831 (112.91)****	0.797064 (107.17)****	0.824853 (140.55)****	0.82963 (159.52)****	0.771257 (158.54)****
USD_RUBLE	-2.24E-06 (-6.43)****	-2.11E-06 (-6.55)****			
USDINDEX	2.72E-06 (3.69)***	2.17E-06 (3.57)***			
LNVEF_USD	3.81E-05 (2.68)**	3.15E-05 (2.08)**			
POUNDUSD	9.00E-05 (1.54)	0.000167 (6.62)****			
YUAN_USD	-5.19E-05 (-1.67)				
RESOLUTION2371			-2.10E-05 (-.3)		
RESOLUTION2375			2.09E-04 (2.14)*		
RESOLUTION2397			0.000341 (1.14)		
FALL_OF_ISIS_MOSUL			3.91E-04 (1.9)		
FALL_OF_ISIS_RAQQA			4.87E-05 (.41)		
BREXIT_VOTE			-8.74E-05 (-4.82)****		
MTGOXCLOSURE				0.000362 (1.99)*	
SILK_ROAD_CLOSE				0.0011 (7.81)****	
BRENTEU					1.50E-06 (11.22)****
_10_YEAR_TREASURY_YIELD					7.62E-05 (11.97)****
FFR					1.64E-05 (1.51)
R-squared	-0.001547	-0.001671	-0.000264	-0.000522	-0.000375
S.D. of d.v.	0.053399	0.053399	0.051183	0.051183	0.051183
S.E. of Reg.	0.053455	0.053458	0.051199	0.051206	0.051202
Schwarz criterion	-3.6793	-3.68289	-3.616953	-3.656494	-3.618733

Table 12 Results of four GARCH (1,1) with different explanatory variables in the variance equation.

Dependent Variable: BTCLOGRETURNS
Method: ML ARCH - Normal distribution (BFGS / Marquardt steps)
Date: 04/15/19 Time: 14:26
Sample (adjusted): 1426 2692
Included observations: 1267 after adjustments
Convergence achieved after 44 iterations
Coefficient covariance computed using outer product of gradients
Presample variance: backcast (parameter = 0.7)
GARCH = C(4) + C(5)*RESID(-1)^2 + C(6)*GARCH(-1) + C(7)*DELTALNGOLD + C(8)*DELTALNETHERIUM

Variable	Coefficient	Std. Error	z-Statistic	Prob.
C	0.001765	0.000927	1.903961	0.0569
DELTALNGOLD	-0.218996	0.103734	-2.111140	0.0348
AR(6)	0.084422	0.030144	2.800617	0.0051
Variance Equation				
C	3.83E-05	4.58E-06	8.380770	0.0000
RESID(-1)^2	0.161905	0.014375	11.26265	0.0000
GARCH(-1)	0.832593	0.011529	72.21906	0.0000
DELTALNGOLD	0.004801	0.001517	3.164880	0.0016
DELTALNETHEREUM	-0.000344	6.71E-05	-5.122035	0.0000

R-squared	0.001274	Mean dependent var	0.002016
Adjusted R-squared	-0.000306	S.D. dependent var	0.039954
S.E. of regression	0.039960	Akaike info criterion	-3.883114
Sum squared resid	2.018359	Schwarz criterion	-3.850631
Log likelihood	2467.953	Hannan-Quinn criter.	-3.870911
Durbin-Watson stat	2.031714		

Table 13 GARCH(1,1) results with explanatory variables of the log first difference of gold price and Ethereum price.

Conclusion

This research reaffirmed that GARCH models are an appropriate tool for modeling price variance in Bitcoin. However, the different specifications of the GARCH model produced mostly inconclusive results at this juncture. The GARCH-M results showed that the inclusion of the conditional variance term and the conditional standard deviation term in the mean equation did not have a statistically significant effect on its value. Both the EGARCH and TGARCH results produced inconclusive results regarding the impact and of the asymmetry for the variance, but confirmed the utility of the EGARCH as a forecasting tool because it greatly improved the significance of the GARCH term in the variance equation. On the basis of this data set, it would seem that the variance of Bitcoin is mostly symmetrical in response to good and bad news. Despite utilizing a wider time window with more daily observations than previous researchers, I was still unable to conclude an alternative viable GARCH specification that definitively fits Bitcoin's variance over time.

The addition of explanatory variables into the variance equation showed that Bitcoin price volatility can be partially explained by explanatory variables that are connected to crude oil. UN Resolution 2375, which imposed heavy crude oil sanctions on North Korea, has a significant and positive coefficient implying that there was increased variance in the month following the implementation of the restrictions. United Nations reports have concluded that North Korea has amassed around $670 million worth of Bitcoin and other currencies through theft and computer hacking, in order to skirt western economic sanctions[xix]. The coefficient on the dummy variable representing the date of the allied capture of the ISIS capital of Mosul is almost significant at the 5% level and has a positive value. Like North Korea, there is evidence that the ISIS government has utilized Bitcoin to accept donations from global sympathizers, as well as make certain payments. The coefficient for the highly statistically significant dates representing the month after the initial Brexit vote implies that there was a reduction in the price variance. This observation makes sense because the intense uncertainty caused investors to redirect their assets toward more stable investments, illustrating market sentiment for Bitcoin. Both closures of the Mt. Gox Bitcoin exchange and the Silk Road Marketplace lead to periods of increased price variance, as the future of the currency was imperiled by the reduction in its utility. Bitcoin variance is positively and significantly related to the price of Brent Crude oil as well as the 10 year treasury yield rate. For the treasury rate, since yields rise when prices fall, this means when there is less demand for safe assets there is also increased price variance in Bitcoin. This relationship also illustrates Bitcoin's current utilization as a high risk speculative investment. Given the risk and historic returns of Bitcoin, this result makes sense. Explaining the relationship with crude oil is more complicated. When Crude prices rise, Bitcoin prices exhibit greater volatility. This dynamic could be due to both securities' sensitivity to other macroeconomic forces or Bitcoins utility to trade oil for countries under economic sanction such as Venezuela or North Korea. Without further investigation, it is impossible to deduce the exact source of this relationship.

Interestingly, assets that are widely considered highly related with Bitcoin produced inconclusive results in the GARCH model. Instead, variables representing the gold price per troy ounce and Ethereum to USD exchange rate had to be transformed into their logarithmic first difference, representing the log of the change in price, to reveal their relationship with Bitcoin. After this transformation is performed, both variables are highly statistically significant and have relatively large coefficients when compared to the other explanatory variables examined in this research. The coefficients of the two assets have opposite signs, so an increasing log price change of gold leads to greater price variance in Bitcoin, and increasing log price change in

Ethereum leads to reduced price variance. This relationship between these assets imply that conditions that lend themselves to gold price volatility, also lend themselves to Bitcoin price volatility. However, conditions that increase Ethereum volatility, reduce Bitcoin volatility because they are competitors. An area for further research would be to investigate the overlap between Bitcoin users and Ethereum users, and if users utilize the cryptocurrencies for different purposes.

When investigating the combined effect of a basket of five currencies from the countries that are responsible for the most Bitcoin traffic, we remove the Chinese Renminbi because it exhibited very little influence on Bitcoin price variance. The Yuan is heavily controlled by the Chinese central bank, and so it is reasonable that the currency exhibits little relation to other market instruments. After removing the Yuan from the variance equation, all currencies have a statistically significant relationship with Bitcoin which is unsurprising because global currencies are highly related due to their use in trade and commerce. However, it is interesting to see that they are all significantly related to Bitcoin because it implies that Bitcoin is thought of and used as a viable currency by global markets. Also, although not robust enough to have an effect within its category of macroeconomic drivers, Bitcoin does exhibit a response to the Federal Funds Rate. This is consistent with previous findings, though in an opposite direction from the relationship reported by Dyrhberg (2016). It is also interesting that Bitcoin and the British pound exhibit the most statistically significant relationship. According to the correlogram in the appendix, the pound and the USDindex exhibit the lowest correlation out of the entire basket. It is uncertain why the pound would have such a notable relationship with Bitcoin variance.

There are many avenues for further research into Bitcoin and its relations and dynamics with the macroeconomy. As the asset becomes more integrated into global markets, these behaviors may continue to develop to maturity, so it is fruitful to continue testing and tracking these changes over time. Different models and different transformations should be tested in further research, including Markov switching models which were untested in this particular research, but have previously yielded great results in modeling Bitcoin volatility. Developing a better understanding of Bitcoins relationships with other commodities and factors will aid investors who could use it as a hedge to avoid exposure in their portfolio. Crucial developmental events in Bitcoins maturity could include acceptance as a payment method for federal taxes, or other forms of adoption and recognition by a country. It may also be rewarding to examine these types of macroeconomic relationships for other cryptocurrencies and understand how they are used differently based on the structures of their underlying software design and capabilities. Cryptocurrencies vary widely in their degrees of anonymity, transaction speed, and scalability, and these variables may yield different products over time. Bitcoin itself may or may not stand the test of time, but the indisputable utility of the blockchain implies that the cryptocurrency as a whole is likely here to stay. As there are over 1600 separate cryptocurrencies[xx], studying what make some more stable and secure than others is of great long term utility. Also, because the issuance of ICO (Initial Coin Offerings) has become a fairly popular and lucrative venture in recent years, there could be enormous profit incentive for engineers that can perfect the most important variables for cryptocurrency software.

Appendix

Proof of GARCH(1,1) as an infinite ARCH(p) process

$$
\begin{gathered}
h_t = \gamma_0 + \delta h_{t-1} + \gamma_1 u_{t-1}^2 \\
= \gamma_0 + \delta(\gamma_0 + \delta h_{t-2} + \gamma_1 u_{t-2}^2) + \gamma_1 u_{t-1}^2 \\
= \gamma_0 + \gamma_1 u_{t-1}^2 + \delta\gamma_0 + \delta^2 h_{t-2} + \delta\gamma_1 u_{t-2}^2 \\
= \gamma_0 + \gamma_1 u_{t-1}^2 + \delta\gamma_0 + \delta^2(\gamma_0 + \delta h_{t-3} + \gamma_1 u_{t-3}^2) + \delta\gamma_1 u_{t-2}^2 \\
\cdots \\
= \frac{\gamma_0}{1-\delta} + \gamma_1(u_{t-1}^2 + \delta u_{t-2}^2 + \delta^2\gamma_1 u_{t-3}^2 + \cdots) \\
= \frac{\gamma_0}{1-\delta} + \gamma_1 \sum_{j=1}^{\infty} \delta^{j-1} u_{t-j}^2
\end{gathered}
$$

	BTCOPEN	EUROUSD	ETHUSD	POUNDUSD	YUAN_USD	USD_RUBLE	USDINDEX	LNVEF_USD
BTCOPEN	1	-0.2120874	0.47983787	-0.2146598	0.20409158	0.51216299	0.6243506	0.72996689
EUROUSD	-0.2120874	1	-0.5288441	0.72908065	-0.6141151	-0.7419786	-0.5571988	-0.5178844
ETHUSD	0.47983787	-0.5288441	1	-0.7959346	0.7913577	0.34646822	0.2140724	0.76255724
POUNDUSD	-0.2146598	0.72908065	-0.7959346	1	-0.8113695	-0.3299144	-0.1218763	-0.6244838
YUAN_USD	0.20409158	-0.6141151	0.7913577	-0.8113695	1	0.29826768	0.1498334	0.42412535
USD_RUBLE	0.51216299	-0.7419786	0.34646822	-0.3299144	0.29826768	1	0.88592916	0.61400928
USDINDEX	0.6243506	-0.5571988	0.2140724	-0.1218763	0.1498334	0.88592916	1	0.53284917
LNVEF_USD	0.72996689	-0.5178844	0.76255724	-0.6244838	0.42412535	0.61400928	0.53284917	1

	BRENTEU	_10_YEAR_TREASURY_YIELD	FFR	GOLD_VALUE
BRENTEU	1	0.079480412	-0.356622398	0.699697724
_10_YEAR_TREASURY_YIELD	0.079480412	1	0.493471652	-0.377691194
FFR	-0.356622398	0.493471652	1	-0.264827083
GOLD_VALUE	0.699697724	-0.377691194	-0.264827083	1

Correlograms

GARCH = C(3) + C(4)*RESID(-1)^2 + C(5)*GARCH(-1) + C(6)*GOLD_VALUE

Variable	Coefficient	Std. Error	z-Statistic	Prob.
C	0.002455	0.003445	0.712641	0.4761
AR(6)	0.005000	0.050283	0.099437	0.9208
Variance Equation				
C	0.002619	0.000658	3.982644	0.0001
RESID(-1)^2	0.150000	0.023984	6.254221	0.0000
GARCH(-1)	0.600000	0.081619	7.351218	0.0000
GOLD_VALUE	0.000000	1.04E-05	0.000000	1.0000

R-squared	0.000410	Mean dependent var	0.002455
Adjusted R-squared	0.000037	S.D. dependent var	0.051183
S.E. of regression	0.051182	Akaike info criterion	-2.790062
Sum squared resid	7.028317	Schwarz criterion	-2.776888
Log likelihood	3751.658	Hannan-Quinn criter.	-2.785297
Durbin-Watson stat	2.048151		

Table 14 The value of gold per troy ounce shows no relationship to Bitcoin variance.

GARCH = C(3) + C(4)*RESID(-1)^2 + C(5)*GARCH(-1) + C(6)*FFR

Variable	Coefficient	Std. Error	z-Statistic	Prob.
C	0.001550	0.000767	2.021025	0.0433
AR(6)	0.084448	0.021491	3.929509	0.0001
Variance Equation				
C	4.87E-05	2.69E-06	18.12356	0.0000
RESID(-1)^2	0.170131	0.007039	24.16832	0.0000
GARCH(-1)	0.829572	0.005833	142.2253	0.0000
FFR	5.07E-06	2.25E-06	2.254936	0.0241

R-squared	-0.000060	Mean dependent var	0.002455
Adjusted R-squared	-0.000432	S.D. dependent var	0.051183
S.E. of regression	0.051194	Akaike info criterion	-3.638327
Sum squared resid	7.031620	Schwarz criterion	-3.625153
Log likelihood	4890.454	Hannan-Quinn criter.	-3.633562
Durbin-Watson stat	2.051047		

Table 15 Independent test of the Federal Funds rate as an explanatory variable in the AR(6) GARCH(1,1) model.

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