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Quantum Stabilization of a Relativistic Degenerate Star Beyond the Chandrasekhar Mass Limit*

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Summary

In contrast to the widely held belief that a degenerate star that has exhausted its nuclear fuel will, if sufficiently massive, unremittingly collapse to a singularity in space (unless the contraction is prevented by some unknown process of quantum gravity acting at the scale of the Planck length), I present a heuristic argument, based on known quantum processes, for the existence of stable equilibrium states of neutron stars and quark stars with macroscopic radii and masses unconstrained by the Chandrasekhar limit. The processes that stabilize the star against gravitational contraction involve strong magnetic coupling of the constituent fermions and fermionic pair production at the expense of gravitational potential energy.

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Although much remains to be learned about the final evolution of stars that have exhausted their nuclear fuel, there is a broad consensus regarding certain basic conclusions. A star whose contraction can be halted by electron degeneracy pressure collapses to form a white dwarf—in effect, an object with approximately the mass of the Sun compacted to the size of a planet. If, however, electron degeneracy pressure cannot achieve hydrostatic equilibrium, the collapse continues until inverse beta decay converts the constituent electrons and protons into neutrons whose degeneracy pressure halts the collapse, creating in effect an object with approximately the mass of the Sun compacted to the size of a city (~10 km). But if neutron degeneracy pressure cannot halt the collapse, then it is believed nothing can spare the star from its subsequent fate: formation of a stellar black hole, i.e. a singularity in the spacetime continuum detectable only through its gravitational field.

The collapse of a neutron star to a black hole, first investigated in detail by Oppenheimer and his colleagues[1], need not be an inescapable conclusion if one takes account of quantum processes outside the framework of classical general relativity. Recent quantum chromodynamical studies of the behavior of nuclear matter at extremely high densities suggest the existence of new states of matter wherein nucleons themselves disrupt, producing a quark-gluon plasma or a quark-gluon condensate.[2,3] These processes entail particle creation and strong fermionic correlations, such as may occur under conditions comparable to those within stars collapsing to densities far beyond those of normal nuclear matter.

Although the behavior of matter subject to extreme nuclear forces and extreme gravity poses a formidable problem for which adequate general relativistic equations of motion have yet to be derived, one can nevertheless gain insights from a heuristic analysis of relativistic quantum particles subject to Newtonian gravity. An argument of this kind was given by Hawking and Ellis[4] to demonstrate that a cold star of *noninteracting* fermions of total mass greater than a certain limit *cannot* reach a stable equilibrium. Their analysis, however, did not take account of the quantum interactions central to this essay, which *do* lead to stable equilibria. Thus, the use of Newtonian gravity, while not as quantitatively rigorous as an analysis based on general relativity, may yet give results that are qualitatively valid under the extended circumstances investigated here, particularly if the predicted size of the resulting stable stars is comparable to the Schwarzschild radius as opposed to the Planck length $\sim 10^{-35}$ m.

Let us begin, therefore, with a gas of fermions of mass m and magnetic moment $\mu = g\left(\frac{e\hbar}{2mc}\right)$ comprising a star of mass M_o and particle number N. The total energy of the star as a function of its radius r takes the form

$$\frac{U_T}{mc^2} = N\left[\left(1+\frac{N^{2/3}\lambda^2}{r^2}\right)^{1/2} - 1\right] + \varepsilon N^2\left[\frac{3g^2\alpha_{fs}}{16\pi}\frac{\lambda^3}{r^3}\right] - \frac{3}{5}\left(\frac{M_o}{m_P}\right)^2\frac{\lambda}{r} \qquad (1)$$

in which the three terms on the right represent respectively the relativistic kinetic energy U_K, the fermion magnetic dipole coupling energy U_M, and the Newtonian gravitational potential energy U_G. The equation is expressed in terms of dimensionless ratios in which $m_P = \sqrt{\hbar c/G}$ is the Planck mass, $\lambda = \hbar/mc$ is the fermion Compton wavelength, and $\alpha_{fs} = e^2/\hbar c$ is the Sommerfeld fine structure constant. For readability and ease of calculation, I employ throughout this paper standard physics units for the universal constant of gravity G = 6.67×10^{-11} Nm^2/kg^2, the speed of light c = 3.0×10^8 m/s, Planck's constant (divided by 2π) $\hbar = 1.055\times10^{-34}$ Js, and the electron charge $e = 4.18\times10^{-10}$ esu. ε is a factor that takes one of the values ±1, 0.

The first term in Eq. (1) derives from the relation $U_K = mc^2\left(\sqrt{1+(p/mc)^2}-1\right)$ in which $p=\hbar/\xi$ is the fermion linear momentum and $\xi = N^{-1/3}\, r$ is the coherence length of the fermion wave function[5]. The second term derives from the magnetic dipole interaction $U_M = \varepsilon N\mu^2/\xi^3$ simplified to pertain to nearest-neighbor interactions in the two cases in which contiguous moments are either aligned ($\varepsilon = +1$) for minimum entropy or antiparallel ($\varepsilon = -1$) for minimum energy. It will be seen shortly that in a degenerate star, for which the fermions are effectively at 0 K, stable equilibrium results from the parallel alignment of magnetic moments. Finally, the third term is the familiar Newtonian gravitational self-energy of a massive sphere of uniform density (an approximation that is convenient but not essential) .

In the absence of the magnetic interaction (ε = 0), the nonrelativistic and relativistic reductions of U_K respectively lead to $N^{5/3}\dfrac{\lambda^2}{2r^2}$ and $N^{4/3}\dfrac{\lambda}{r}$, which correspond precisely to the nonrelativistic and relativistic Fermi energy.[6] Since the radial dependence of the relativistic Fermi energy is the same as that of the gravitational term U_G, the minimization of U_T (for constant M_0 and N) leads to a result independent of radius (or, equivalently, independent of density) and therefore to an upper limit (Chandrasekhar limit) of the mass M_0 of a relativistic degenerate star, as is well known.

However, with inclusion of the magnetic interaction, the energy U_T in the relativistic limit of U_K takes the form $U_T = \dfrac{a}{r}+\dfrac{b}{r^3}$, leading to a real-valued, stable equilibrium radius $r_{eq} = \sqrt{-3b/a}$ under the conditions $a < 0,\ b > 0$, which requires that $\varepsilon = +1$ in Eq. (1). Expressed in terms of the stellar parameters, the equilibrium radius is

$$r_{eq} = N\lambda\left[\frac{\beta}{\gamma(M_{\circ}/m_{P})^{2} - N^{4/3}}\right]^{1/2} \tag{2a}$$

where $\beta = 9g^{2}\alpha_{fs}/16\pi$ and $\gamma = 3/5$. Expressing the mass of the star in terms of the solar mass ($M_{\circ} = \sigma M_{\ast}$) and the number of fermions in terms of the number of neutrons in a solar-mass neutron star ($N = \tau(M_{\circ}/m_{n}) = \sigma\tau\,(M_{\ast}/m_{n})$), reduces Eq. (2a) to the form

$$r_{eq}/\lambda = \frac{(1.2\times10^{18})\tau}{\sqrt{1-2.52\left(\tau^{2}/\sigma\right)^{2/3}}}\,. \tag{2b}$$

As seen from Eq. (2a), stable stars result with masses *greater* (not less) than a minimum threshold: $M_{\circ} > \gamma^{-1/2}N^{2/3}m_{P}$. Note, too, that the mass of the fermionic particle comprising the star appears only in the Compton wavelength and therefore does not affect the stability criteria.

Applied to the case of a fully spin-aligned neutron star ($m_{n} = 1.67\times10^{-27}$ kg, $\lambda_{n} = 2.1\times10^{-16}$ m, $g_{n} = -1.91$, $\tau = 1$), Eq. (2b) leads to stellar radii (in meters) $r_{eq} = \dfrac{244.8}{\sqrt{1-2.52\sigma^{-2/3}}}$ and a threshold mass parameter $\sigma > 4$, i.e. a minimum of 4 solar masses. For $\sigma = 5$, the star has a radius of ~0.3 km and a density of $\sim 8.7\times10^{19}$ g/cm^{3}. If, however, under the conditions of extreme gravitational compression, the nuclei dissolve into bare quarks ($\tau = 3$) which, for illustration, are here taken to be s quarks (m_{s}~100 MeV/c^{2} $\sim \frac{1}{10}m_{n}$, $\lambda_{s} = 2.1\times10^{-15}$ m g_{s}~ -0.6)[7], the equilibrium radius of the spin-aligned quark star is $r_{eq} = \dfrac{2.3\times10^{3}}{\sqrt{1-10.9\sigma^{-2/3}}}$, and the minimum threshold mass parameter is $\sigma > 36$.

For $\sigma = 40$, a star with radius ~8.8 km and density $\sim 2.8\times10^{16}$ g/cm^3 results. Such end products of stellar evolution, if they exist, would be cold, highly magnetized, degenerate systems like neutron stars, but with masses greater than the currently accepted Chandrasekhar mass limit ($\sigma \sim 1.4$ for white dwarfs and $\sigma \sim 0.7$ for neutron stars[8]) and radii that can fall below the corresponding Schwarzschild radius. It must be born in mind, however, that the model presented is a heuristic one, and that in a fully general relativistic calculation the star may be stabilized at a size greater than the Schwarzschild radius. Also, in some successful future merger of quantum theory and gravity, the very concept of the Schwarzschild radius may be modified if not eliminated.

In the process of collapse, a degenerate neutron star unsustained by neutron degeneracy pressure, and without long-range magnetic order to create magnetic pressure, will eventually reach a critical radius r_c such that the gravitational energy $-\Delta U_G$ released into a spherical volume V equals the energy density $2mc^2/\lambda^3$ at which fermion pair production becomes conceivable. The particle number N is then no longer constant., and the general relativistic analyses of neutron star collapse by Oppenheimer, Hawking, and others are no longer valid. From the relation $-\Delta U_G/V = 2mc^2/\lambda^3$ and the assumption that r_c is much less than the initial stellar radius, it follows that the critical radius (in meters) is

$$\frac{r_c}{\lambda} = \left(\frac{9}{40\pi}\right)^{1/4} (M_\odot/m_p)^{1/2} = 1.04\times10^3\sigma^{1/2}\tau. \qquad (3)$$

For a solar-mass neutron star, Eq. (3) yields $r_c \sim 1$ km. The production of neutron-antineutron pairs (or quark-antiquark pairs) does not violate baryon number conservation, but it is unclear whether this law even holds under such extreme conditions. Nonconservation of baryons is already believed to have occurred in the early universe[9].

Without the magnetic interaction in Eq. (1), minimization of the total energy (subject to constant M_0) in the relativistic limit leads to the following implicit relation for the equilibrium radius

$$r_{eq} = \frac{\gamma (M_0/m_P)^2 - N(r_{eq})^{4/3}}{\frac{4}{3} N(r_{eq})^{1/3} (-dN/dr)_{r_{eq}}} . \tag{4}$$

The equilibrium is stable for $(dN/dr)_{r=r_{eq}} < 0$ (condition that particle creation increases as the star contracts) and $(d^2N/dr^2)_{r=r_{eq}} > 0$. It therefore follows from Eq. (4) that $M_0 > \gamma^{-1/2} N(r_{eq})^{2/3} m_P$. Under the assumption that the gravitational energy released in contraction from r_c to r_{eq} goes entirely into fermion pair production, the additional number of particles at r_c is $\gamma(M_0/m_P)^2 \lambda \left(r_{eq}^{-1} - r_c^{-1}\right)$, and Eq. (4), applied to a neutron star, results to good approximation in the inequality $1 > (r_{eq}/r_c) > 0.412 + 1.175\sigma^{-1/2}$ where $\sigma > 4$, as calculated previously.

In conclusion, strong magnetic correlation among fermions (such as would be expected in a neutron star or quark star) and gravitationally induced particle creation (such as is believed to have occurred shortly following the origin of the universe) can serve to brake the relentless contraction of a degenerate star when neutron degeneracy pressure is insufficient. If more rigorous calculation substantiates the argument presented here, then stellar black holes as they are currently construed—either a singularity in spacetime or a mass concentration of Planck length size ($\sim 10^{-35}$m)—do not likely exist, and stellar objects presently so classified will be stable systems of neutrons or quarks (together with gluons) at densities that can exceed 10^{16} g/cm^3 within a radius on the order of, or conceivably inferior to, the Schwarzschild radius.

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