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Mark P. Silverman
Trinity College, mark.silverman@trincoll.edu

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NEW QUANTUM EFFECT OF CONFINED MAGNETIC FLUX ON ELECTRONS

M. P. Silverman*

Advanced Research Laboratory, Hitachi Ltd, Kokubunji, Tokyo 185, Japan

ABSTRACT

A charged particle analogue of the Hanbury Brown-Twiss experiments with photons is described wherein the correlation of electron intensity at two detectors is modulated by magnetic flux confined to a region from which the electrons are excluded. The experimental conditions differ substantially from those of the Aharonov-Bohm effect.

*Visiting Senior Researcher; permanent address: Department of Physics, Trinity College, Hartford, Connecticut 06106 USA

In this paper is reported a novel influence of confined magnetic flux on charged particles different from the well-known and experimentally substantiated Aharonov-Bohm (AB) effect.[1] As commonly understood, the AB effect on unbound charged particles interacting locally with electromagnetic potentials, but not electromagnetic fields, is manifested by a magnetic (or electric) flux dependent shift in the interference fringes (but not envelope[2]) of a particle diffraction pattern. The phenomenon to be described differs from the AB effect in several important ways. First, the influence of a local vector potential is sought, <u>not</u> in the interference of particle probability amplitudes at a single detector, but rather in the correlation of particle fluxes (intensities) at two detectors. Second, whereas the AB effect can be in principle--and, in fact, has been--experimentally demonstrated with single particle wave packets[3], the proposed intensity correlation effect requires wave packets of two or more particles and would therefore give a null result under the conditions so far employed to observe the AB effect on beams.

In analogy with optical processes, the AB effect resembles a magnetic (or electric) flux dependent Young's two-slit experiment in that the resulting interference pattern is characterized by the first order correlation function of the particle field. The effect reported here is a charged particle, magnetic flux dependent analogue of the Hanbury Brown-Twiss (HBT) experiments[4,5], the results of which are characterized by the second order correlation function. There are profound differences, however, in the expected intensity correlation of charged particles, in particular electrons, compared to that of light. The photon is a neutral boson; having no charge it cannot couple directly to a local external vector potential field.[6] Thus, the intensity correlation of light beams at two dectectors can not be affected by the presence of an AB solenoid in an experimental configuration such as is illustrated in Figure 1. Moreover, the requirement of symmetry under particle exchange permits construction of photon states that lead to bunching (chaotic light[5]), no bunching (coherent or laser light[7]), or antibunching (resonance fluorescence from single atoms[8]). Electrons are charged fermions. The "minimal coupling" scheme, which follows from the requirement of gauge invariance, results in a magnetic flux dependent intensity correlation for the configuration of Figure 1, as will be demonstrated. To my knowledge, no such possibility has been considered before in the extensive literature of quantum effects of electromagnetic fluxes.[9] The requirement of antisymmetry under particle exchange leads to electron antibunching irrespective of the coherence properties of the electron beam. The possibility of electron antibunching is implicitly contained in the Fermi-Dirac expression for particle density fluctuations as pointed out briefly by Purcell.[10] This point is developed explicitly and in a more general context here.

Figure 1 shows one schematic experimental configuration by which means the predicted magnetic flux dependence and electron antibunching may be demonstrated. A beam of electrons produced by source S is incident upon a partition with two apertures S_1, S_2 which split the beam by wavefront division; the two components, passing around, but not through, a solenoid with confined magnetic flux, illuminate two detectors D_1, D_2 whose instantaneous outputs are sent to a correlator C. The correlation procedure has been described in detail for photons[11] and massive particles[12].

The correlation of electron intensities at detectors D_1, D_2 at times t_1, t_2, respectively, is given by the (unnormalized) second order correlation function

$$G^{(2)}(D_1,t_1;D_2,t_2) = \mathrm{Tr}\left\{\rho\left(\phi^\dagger(D_1,t_1)\ \phi^\dagger(D_2,t_2)\ \phi(D_2,t_2)\ \phi(D_1,t_1)\right)\right\} \tag{1}$$

where ρ is the density matrix characterizing the electron field of the source. The second-quantized flux operator $\phi(D_j,t_j)$ $(j = 1,2)$ is a linear superposition of contributions from apertures S_1 and S_2

$$\phi(D_j,t_j) = \left[\phi_1(D_j,t_j) + \phi_2(D_j,t_j)\right]/\sqrt{2} \tag{2a}$$

In the absence of a vector potential field, $\phi_i(D_j,t_j)$ $(i = 1,2)$ may be expressed in the form[12]

$$\phi_i(D_j,t_j) \equiv \phi_i^\circ(D_j,t_j) = \sum_{k,s} \sqrt{k}\ u_{ks}^{(i)}(D_j)\ \exp(-iw_k t_j)\, b_{ks}^{(i)} \quad . \tag{2b}$$

The factor $\sqrt{k}$ converts the standard field operator[13] into a flux operator in order that Eq. (1) may represent an intensity, rather than density, correlation. The mode function $u_{ks}^{(i)}$ is frequently taken to be a plane or spherical wave, but will here be left unspecified. For nonrelativistic particles of mass m, the energy (in units of $\hbar = c = 1$) is $w_k = k^2/2m$. The operators $b_{ks}^{(i)}$, $b_{ks}^{(i)\dagger}$ respectively annihilate and create an electron with momentum k and spin projection s in the field that has emerged from aperture S_i. For apertures of equal size, one can write

$$b_{ks} = (b_{ks}^{(1)} + b_{ks}^{(2)})/\sqrt{2} \quad \text{(and h.c.)} \tag{3a}$$

where b_{ks}, $b_{ks}^\dagger$ respectively annihilate and create the corresponding electron in the beam incident upon the partition. All operators satisfy the standard anticommutation relations

$$\left\{b_{ks}, b_{k's'}^\dagger\right\} = \delta_{kk'}\ \delta_{ss'} \quad ; \quad \left\{b_{ks}, b_{k's'}\right\} = 0 \ \text{(and h.c.)} \tag{3b}$$

$$\left\{b_{ks}^{(i)}, b_{k's'}^{(j)\dagger}\right\} = \delta_{ij}\ \delta_{kk'}\ \delta_{ss'} \quad ; \quad \left\{b_{ks}^{(i)}, b_{k's'}^{(j)\dagger}\right\} = 0 \ \text{(and h.c.)} \quad . \tag{3c}$$

In the presence of a vector potential each single-particle wave function from which is constructed the antisymmetric multiparticle wave function or density matrix of the electron beam incurs a gauge-dependent phase factor. In the second quantized formalism this leads to the flux operators

$$\phi_i(D_j, t_j) = \phi_i^{\circ}(D_j, t_j) \exp(-i\theta_{ij}) \tag{4a}$$

where

$$\theta_{ij} = q\left(\int_S^{D_j} A ds\right)_{\text{via } S_i} \quad ; \tag{4b}$$

q is the electron charge; the integration path is from S to D_j through S_i. Finally, to allow for a (possibly random) phase shift δ in the transmissivity of aperture S_2 relative to that of S_1, one multiplies the flux operator ϕ_2 by $\exp(i\delta)$.

Evaluation of Eq. (1) leads to an expression of the form

$$G^{(2)}(D_1, t_1; D_2, t_2) = A_o + A_1\cos(q\phi_m + a_1 + \delta) + A_2\cos\{2(q\phi_m + a_2 + \delta)\} \tag{5a}$$

where

$$A_o = \left[(11|11) + (22|22) + (12|21) + (21|12) + 2\mathrm{Re}\,(12|12)\right]/4 \tag{5b}$$

$$A_1\cos(q\Phi_m + a_1 + \delta) = 2\,\mathrm{Re}\left(\left[(11|12) + (11|21) + (12|22) + (21|22)\right]e^{(i(q\Phi_m + \delta))}\right) \tag{5c}$$

$$A_2 = |(11|22)|/2 \tag{5d}$$

$$2a_2 = \arg\{(11|22)\} \tag{5e}$$

and

$$(ij|kn) = \mathrm{Tr}\left\{\rho(\phi_i^{\circ\dagger}(D_1, t_1)\ \phi_j^{\circ\dagger}(D_2, t_2)\ \phi_k^{\circ}(D_2, t_2)\ \phi_n^{\circ}(D_1, t_1)\right\} \quad . \tag{5f}$$

The magnetic flux ϕ_m within the solenoid is given by

$$q\emptyset_m = q\oint A ds \quad = \theta_{11} - \theta_{21} = \theta_{12} - \theta_{22} \quad . \tag{5g}$$

It is of interest to record, for purposes of comparison, the corresponding relation for the AB effect which is given by the first order correlation function at one detector D at time t

$$G^{(1)}(D,t) = \mathrm{Tr}\left\{\rho\, \phi^{\dagger}(D,t)\ \phi(D,t)\right\} \quad . \tag{6a}$$

Eq. (6a) reduces to

$$G^{(1)}(D,t) = B_o + B_1 \cos(q\emptyset_m + b_1 + \delta) \tag{6b}$$

where

$$B_o = (1|1) + (2|2) \tag{6c}$$

$$B_1 = 2|(1|2)| \tag{6d}$$

$$b_1 = \arg\left\{(1|2)\right\} \tag{6e}$$

and

$$(i|j) = \mathrm{Tr}\left\{\rho\, (\phi_i^{o\dagger}(D,t)\ \phi_j^{o}(D,t)\right\} \tag{6f}$$

From Eqs. (5) and (6a) there follow several important consequences. The intensity correlation $G^{(2)}$ is a harmonic function of $q\emptyset_m$ and $2q\emptyset_m$; the AB effect, represented by $G^{(1)}$, is a harmonic function of $q\emptyset_m$. For random phase δ, which makes S_1 and S_2 behave as two independent incoherent sources, the flux dependence vanishes from both $G^{(2)}$ and $G^{(1)}$. In that case, $G^{(1)}$ is a sum of intensities from each aperture and shows no quantum interference. In $G^{(2)}$, however, the surviving term A_o exhibits interference effects in the nondiagonal matrix elements (last three terms). This preservation of phase information makes intensity correlation interferometry a useful technique for determining scattering amplitudes.[11,12]

The detailed space-time behavior of $G^{(2)}$ depends on the matrix elements constituting coefficients A_o, A_1, A_2 which, in turn, depend on the characteristics of the beam. Two points hold generally, however. First, $G^{(2)}$ vanishes identically at all space-time points for any electron field consisting exclusively of single-particle wave packets, since each matrix element has two annihilation operators. This is well-known in the case of intensity correlation of light[14] and corresponds to the fact that detection of a particle at one detector precludes detection at the other detector; a nonvanishing correlation at two detectors requires at least two particles. (This nontrivial result lies at the root of the discrepancies between classical wave and quantum determinations of second order correlations.[14]) Second, it follows from the antisymmetry of electron wave functions that $G^{(2)}$ vanishes identically for the case of instantaneous correlation ($t_1 = t_2$) of spin-polarized electrons at two contiguous detectors equidistant from both apertures. This example of electron antibunching may be seen as follows. Each matrix element $(ij|kl)$ leads to elements of the form $\langle\phi|b_{k_1s}^{(i)\dagger}\, b_{k_2s}^{(j)\dagger}\, b_{k_3s}^{(k)}\, b_{k_4s}^{(l)}|\phi\rangle$ where $|\phi\rangle$ is a particular n-particle, spin-polarized state contributing to the density matrix. The element is nonvanishing for two sets of momentum equalities, $(k_1 = k_3,\, k_2 = k_4)$ and $(k_1 = k_4,\, k_2 = k_3)$, which, because of the anticommutation relations, lead to functions with opposite signs. Since the mode functions $u_{ks}^{(1)}$, $u_{ks}^{(2)}$ for the specified experimental configuration have equal values at both detectors, the two functions cancel one another when summed. The individual terms $(ij|kl)$, and hence $G^{(2)}$, are null.

In conclusion, it has been shown that the local interaction of electrons with a vector potential in a region free of magnetic and electric fields can affect the correlation of electron intensity at two detectors. The second order correlation function of the electron field manifests electron antibunching for both coherent and incoherent sources. The influence of the confined magnetic flux on the particle flux correlation occurs only for a coherent source; the effect appears as a modulation of the second order correlation function which still manifests electron antibunching.

The objective of this paper has been to demonstrate in general terms the theoretical existence of the above effects. Detailed analysis of particular experimental configurations and the feasibility of producing the effects with current technology, and extension to charged bosons will be presented elsewhere.[15]

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FIGURE CAPTION (Figure 1)

Schematic experimental configuration. An electron beam produced by source S passes through apertures S_1 and S_2 and illuminates detectors D_1, D_2 whose outputs are sent to correlator C and thence to recorder R. The second order correlation function is sensitive to the magnetic flux ϕ_m confined to the solenoid interior.

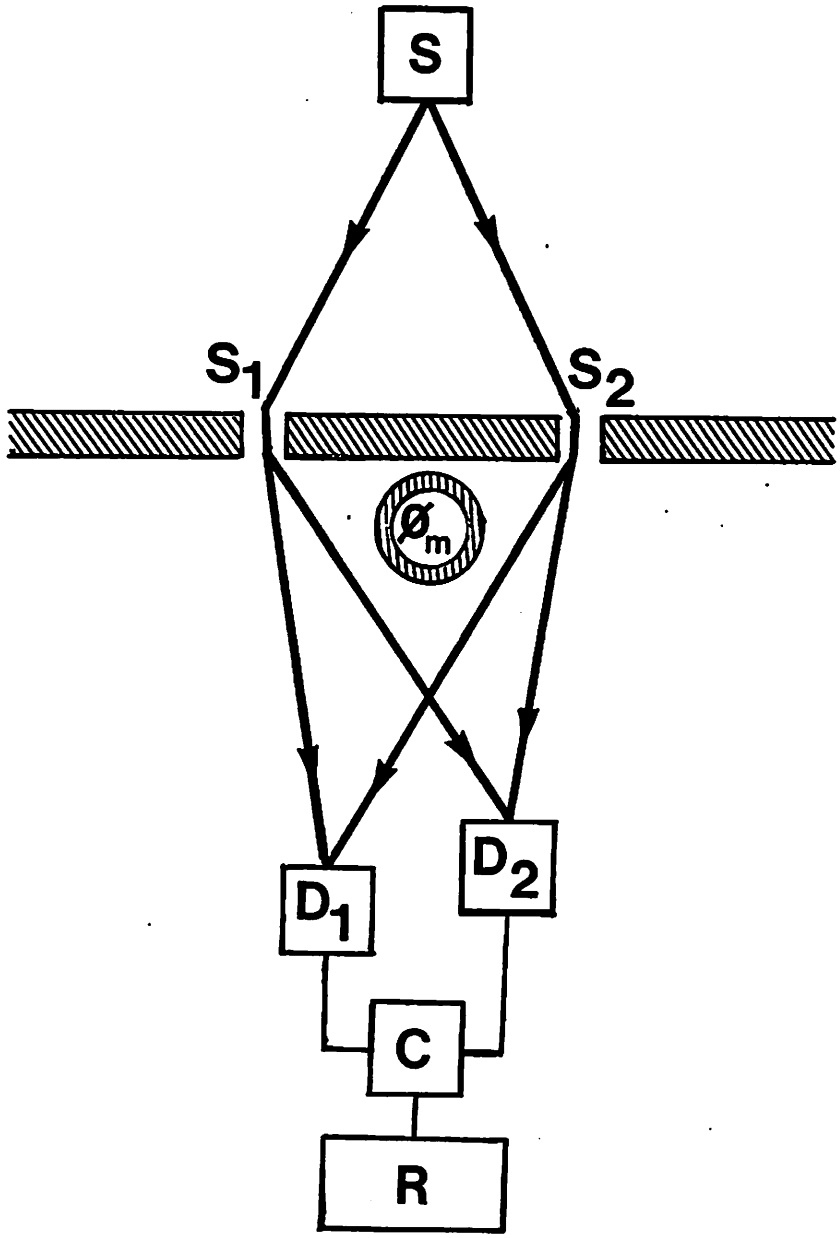

Fig. 1