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Writing in the Disciplines: How Math Fits Into the Equation [post-print]

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FEATURE ARTICLE

Writing in the Disciplines: How Math Fits in the Equation

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Teaser Text:

Understanding the types of and purposes for elementary mathematical writing can provide a framework to guide mathematical writing instruction.

Abstract:

Writing is an important mode of thinking and learning for elementary students. Consistent efforts have been made to encourage discipline-specific writing, yet defining qualities of elementary mathematical writing have historically been underdeveloped. This article offers educators a new framework that conceptualizes mathematical writing as writing to reason and to communicate mathematically. Specifically, the framework defines four types of elementary mathematical writing: exploratory, informative/explanatory, argumentative, and mathematically creative. The authors explain and explore these types and their associated purposes through classroom vignettes. Informed by existing practices in mathematics and writing, strategies are offered to support teachers with the implementation of mathematical writing.

Keywords:

Content literacy, Writing, Theoretical perspectives, Specific subject areas (math, art, etc.) < Content literacy, Text types, text features < Content literacy, Writing strategies < Strategies, methods, and materials, Audience < Writing, Modes < Writing, Purpose < Writing, Writing across the curriculum < Writing, Writing to learn < Writing, Vygotskian < Theoretical perspectives

Level of Learners:

2-Childhood, 3-Early adolescence

PAUSE AND PONDER

- Why engage students in discipline-specific writing?
- How do mathematicians use writing to reason and to communicate?
- How can writing help students learn and think mathematically?
- How can teachers begin to create opportunities for students to engage in mathematical writing?

> Writing represents a unique mode of learning—not merely valuable, not merely special but unique. (Emig, 1977, p. 122)

Educators realizing the power of writing have long called for writing in the disciplines (Herrington, 1981; Vacca & Vacca, 2000). Recently, these calls have been made in the Common Core State Standards for English Language Arts (National Governors Association Center for Best Practices [NGA Center] & Council of Chief State School Officers [CCSSO], 2010a). The

Common Core recommends that beginning in grade 3, students need to write "for a range of discipline-specific tasks, purposes, and audiences" (p. 21).

In mathematics, it is notable that Vygotsky (1978) long hypothesized that it was the need to record the notion of quantity that gave rise to written language in the first place. However, as Fortescue (1994) acknowledged two decades ago, and a problem that still persists today (Casa et al., 2016), the tools and resources to support writing in the mathematics discipline are lacking. To address this problem, the Elementary Mathematical Writing Task Force developed a framework to support students' written communication of and reasoning about mathematics (Casa et al., 2016). In this article, we share the types of and purposes for elementary mathematical writing identified by the task force. First, we describe discipline-specific writing to position writing in a mathematics class within the larger context of the writing-to-learn movement (Fulwiler & Young, 1982). Then, we illustrate each type of mathematical writing through classroom vignettes and conclude the article with strategies for implementing writing in the mathematics class.

Writing and Thinking

When we hear the name Vygotsky (1934/1986, 1978), many of us think of the zone of proximal development. You might remember a discussion during your preservice days about scaffolding and how students need challenges that, like in the Goldilocks story, are "just right" to grow and learn. Vygotsky's idea of the zone of proximal development was just one part of a robust model for explaining how children develop conceptual understandings and higher order thinking skills. Language, according to Vygotsky, is an integral tool in building one's ability to think. Students are empowered to build ideas with greater precision if they routinely use a full range of language tools, including writing.

Vygotsky (1978) argued that human language is a mental tool that helps us exert creative control over ourselves and the world around us. Words help humans construct concepts and thoughts, and these ideas allow us to reflect on the past, plan for the future, and exert control over the present. However, when language stays internal to the learner, ideas remain vague and abstract. The more external the language, the more precise the concepts are; the brain is forced to analyze and clarify which aspects are unique to the situation at hand to use the tool of language for interpersonal communication. Thus, written language allows us to visually and symbolically

represent abstract ideas. Writing can also help clarify both preliminary conceptions and misconceptions, as well as well-developed ideas in application to new contexts.

As you read the following classroom vignette about Ms. Sanchez, a hypothetical third-grade teacher, reflect on ways that writing may help support students' understanding of fractions:

> Ms. Sanchez introduced her students to the comparison of fractions. During an initial lesson, she explained to her students why one half is greater than one fourth. She then went through a few more examples. At the end of the lesson, she asked her students if they understood, and all heads nodded affirmatively. She asked if they had any questions, but none of her students raised their hands.
>
> Each student then completed a brief exit ticket, simply circling the appropriate symbol to indicate whether they thought one half was less than, greater than, or equal to one eighth. Later that afternoon, Ms. Sanchez noticed that many of her students incorrectly identified one eighth as greater than one half. She wondered how she could have identified her students' underlying misconceptions earlier and how she can help her students develop their understanding at a deeper level.

To help Ms. Sanchez better understand her students' thinking, she could have engaged them in oral or written discourse. By eliciting student thinking through a discussion, Ms. Sanchez could have gathered information about the whole class's understanding of the comparison of fractions. If Ms. Sanchez had asked her students to write explanations to convey their thinking, she would have seen evidence of each student's conceptual understanding. By simply asking students to select a symbol, the amount of information Ms. Sanchez gathered about each student's understanding was limited.

Writing in the Disciplines

Writing is just as integral to thinking in mathematics as it is to thinking in language arts, social studies, or science, but the purposes for writing are not the same in each discipline. Writing activities and strategies cannot be uniformly transposed from English language arts into other disciplines because our discipline-based thinking goals must drive the structure of our writing activities (Chandler-Olcott et al., 2015; Moje, 2015). This perspective on disciplinary writing "recognizes that each discipline has its own unique language conventions, format, and structure" (Wells, 2018, para. 4).

To support students' learning in various academic discourses, they need writing experiences that align with the specific disciplines. For example, to think and write like scientists, students need "to assert and defend claims in science, demonstrate what they know about a concept, and convey what they have experienced, imagined, thought, and learned" (NGSS Lead States, 2013, p. 1).

To think and write like mathematicians, students can use writing to make sense of problems, to describe and explain, to construct and evaluate arguments (National Council of Teachers of Mathematics [NCTM], 2000; NGA Center & CCSSO, 2010b), and to elaborate on mathematical ideas and discoveries (Burton & Morgan, 2000). Mathematical writing can provide students with an additional medium beyond oral discourse to communicate in mathematics (Baxter, Woodward, & Olson, 2005). Cohen, Casa, Miller, and Firmender (2015) found that second-grade students who engaged in a curriculum emphasizing oral and written discourse demonstrated stronger reasoning and used mathematically precise vocabulary in writing. Additionally, Jurdak and Zein (1998) found that writing had cognitive benefits for students' conceptual understanding and procedural knowledge of mathematics.

The Curriculum and Evaluation Standards for School Mathematics (NCTM, 1989) suggested ways that teachers could provide opportunities for students to write in mathematics class, such as writing a letter to a friend and writing in a journal. Reflecting these recommendations, journal writing has often been a focus in studies of writing in mathematics class. For example, Brown (1997) studied first graders' use of journals to record instances of mathematics that happened outside of the classroom. Kostos and Shin (2010) found that using a mathematical journal improved second-grade students' communication of mathematical thinking.

Yet, many of the attempts at writing in mathematics classrooms reflect an overall lack of clarity about what constitutes mathematical writing (Bossé & Faulconer, 2008). This often results in writing that fails to include these hallmarks of mathematical thinking and practice and that might better be described as writing *about* math. For example, students have been asked to "describe how you feel about solving [this mathematics] problem" (Baxter, Woodward, Olson, & Robyns, 2002, p. 54) or to "write a letter to me telling how you feel about yourself and math class" (DiPillo, Sovchik, & Moss, 1997, p. 309).

Although these questions can engage students in writing, their purpose is not situated in the mathematics discipline. This reflects the need for educators to better understand the notion of mathematical writing, including its types and purposes, as well as strategies for supporting its implementation in the elementary classroom.

The Elementary Mathematical Writing Task Force

To work toward defining elementary mathematical writing, the Elementary Mathematical Writing Task Force met in the fall of 2015. The task force included elementary school and university-based professionals with expertise in mathematics education, writing education, English language learners, students with learning difficulties, and students identified as gifted. The task force identified the overarching goals of mathematical writing as writing to reason and to communicate mathematically, and they recommended four types with associated purposes (see Figure 1). The following vignettes and explanations are intended to help illustrate each of the types and purposes of mathematical writing as they might be used in an elementary classroom.

[Figure 1.]

Types of Mathematical Writing

Exploratory Writing

Yesterday, Ms. Sanchez continued to work on the concept of fractions with her third graders. Her objective was to have her students describe a fraction. Today, she built on that by asking students to respond to the question, "Why are fractions used?" To explore this question, Ms. Sanchez provided students with manipulatives and invited them to write in their journals or talk to a peer. Tasha and Adam quickly grabbed their journals to explore this question, some students found a friend to discuss it, and a few others worked independently using manipulatives.

Adam wrote about how he loves pizza and that fractions can be used to describe a part of the pizza that he ate. Tasha really likes graham crackers, so she jumped right in and showed Adam her writing about how she can break a graham cracker into many pieces and use fractions to describe how many of those pieces she ate.

While comparing their writing, both Adam and Tasha realized that a whole does not have to be the same thing and that the same fraction could represent completely different-sized quantities; if Adam ate one half of a pizza, he would be full, but he'd still be hungry if he ate one half of Tasha's graham cracker. Both referred to their own writing to discuss their thoughts, and their interaction informed each other's thinking about Ms. Sanchez's question.

The type of writing that Adam and Tasha engaged in is exploratory writing (Casa et al., 2016). It is a way for students to make sense of mathematical concepts, problems, and their own ideas. As demonstrated in the vignette, exploratory writing is student initiated and completed for the student to increase his or her understanding. Ms. Sanchez provided students with a question and materials to explore this idea, but she did not direct students on how to explore the concept of fractions. Adam and Tasha decided on their own that writing would be the best way for them to further their understanding.

The audience for this type of writing is generally the student. Thus, exploratory writing may also include other representations such as graphs, diagrams, and/or computations that help the student make sense of the mathematics. Students can use exploratory writing to work out confusions and misconceptions and to document ideas, patterns, or observations. Creating space and time for exploratory writing, like Ms. Sanchez did, can provide students with an opportunity to express and develop their thinking about different mathematical concepts.

Exploratory writing reflects formative understanding, so it should not be used for summative evaluation. However, it can offer valuable insights into students' developing conceptual understandings. This type of writing provides students with opportunities to explore concepts, conjectures, and develop ideas. To encourage this, it is important that students feel comfortable writing and representing their ideas; therefore, grading this writing would likely hamper students' freedom. Exploratory writing can also further enhance whole-class discussions by providing students with an opportunity to first formulate their individual ideas before sharing them. Finally, exploratory writing is an organic process that may look different for each student.

Informative/Explanatory Writing

As Ms. Sanchez walked around observing her students, she noticed that Adam and Tasha had both discovered important concepts about fractions: the whole does not have to be the same size, and the same fraction can represent quantities of different sizes. Curious as to how the students in her class understood this concept, Ms. Sanchez asked the class to compare an estimate of one third of the area of the playground with one third of the (smaller) blacktop.

To support students' reasoning of this concept, Ms. Sanchez decided to have her students write a response to the following prompt: "Explain how you know that one third of the area of the playground is larger than one third of the area of the blacktop." Before writing, Ms. Sanchez led all of her students onto the school grounds to let the students discover which was larger through an active and hands-on experience. The students used a variety of methods to measure one third

of the area of the playground and the blacktop to prepare for writing their explanation. For example, one group of students laid down side-by-side using their bodies as measurement tools. Another group of students used a more traditional method and brought yardsticks from the classroom.

Once back in the classroom, the students took out paper and pencils and wrote their explanations. From the student responses, Ms. Sanchez was able to assess that her students were beginning to realize that the same fraction could represent different-sized quantities depending on the size of the whole.

In this vignette, Ms. Sanchez engaged her students in informative/explanatory writing that "serves to have elementary students provide information or explanations related to mathematical concepts" (Casa et al., 2016, p. 9). This type of writing has two purposes: writing mathematically to describe and to explain. This vignette demonstrates students writing mathematically to explain; the students are writing how they know that one third of the area of the playground is larger than one third of the area of the blacktop. This type of writing can help students clarify their ideas, communicate with precision, and give permanence to their ideas.

The audience for this writing is typically another student or the teacher but may include others. For example, older students may write to younger ones explaining the difference between a circle and a sphere. Similarly, they may write to a local garden supplier to communicate how many bags of soil they have calculated is needed for the school garden. Because of the difference in audience from exploratory writing, students need to attend to how they are writing their ideas to ensure that others can understand their thinking. Compared with exploratory writing, informative/explanatory writing in mathematics often needs to include greater specificity and detail, including appropriate use of mathematical terms, symbols, units, and conventions.

Writing mathematically to describe encourages students to reason about a mathematical concept or idea (Casa et al., 2016). This may include recording observations, describing the attributes of a shape, and/or writing a definition. Writing mathematically to explain includes engaging students in writing about concepts, strategies, and comparisons. For example, students may write to explain the difference between a cube and a square pyramid or explain why they chose a particular strategy for solving a two-digit subtraction problem.

It is important to note that the purpose of writing mathematically to explain demands students move beyond simply listing their steps for solving a problem. For example, instead of

asking a student to describe how she solved 29 + 37, she could be asked to explain (if using the traditional algorithm) the meaning of the 1 recorded above the 2 and 3. This type of writing moves students to demonstrate a conceptual understanding of the algorithm or strategy for solving a problem as opposed to listing (in words and/or numbers) their computational steps.

Informative/explanatory writing is a primary emphasis of the Common Core State Standards (NGA Center & CCSSO, 2010a), calling for students to "write informative/explanatory texts to examine and convey complex ideas and information clearly and accurately through the effective selection, organization, and analysis of content" (p. 18). In developing the types of and purposes for mathematical writing, the task force emphasized the mathematical purpose for this type of writing while also drawing connections to other types of informative/explanatory writing outside of the mathematics discipline (Casa et al., 2016).

Argumentative Writing

Ms. Sanchez read through the student responses and was encouraged by their descriptions of one third of the area of the playground and of the blacktop, but she wanted to help her students recognize that the size of the whole is important to understanding the relative size of a fractional part. To entice students, she secretly brought to school chocolate bars of two different sizes: king size and minis.

Ms. Sanchez gathered her students on the rug and asked them to raise their hand if they thought that a whole chocolate bar was always greater than half of a chocolate bar. All but two students thought a whole chocolate bar would always be greater than half of a chocolate bar. Ms. Sanchez handed out mini Hershey bars to these students. Each of the two students who indicated that this statement would not always be true received half of a king-sized Hershey bar. The students with the mini chocolate bars looked on with envy!

After munching on the Hershey bars and moving back to their desks, Ms. Sanchez posed this claim as a written prompt: "When comparing two different items, the whole of one item is always larger than a half of another item. Do you agree or disagree, and why?" Students eagerly took to writing their responses.

One student, Maya, wrote, "I disagree with Ms. Sanchez's claim because a half can be larger than a whole depending on the size of the whole. For example, if you had a whole puddle and a half of an ocean, the half of the ocean would be larger." Another student, Oscar, wrote, "Ms. Sanchez's claim is sometimes right and sometimes wrong. A half can be smaller than the whole if the wholes of the two things you have are the same. For example, half of my backyard is smaller than my whole backyard."

In reading through her students' writing Ms. Sanchez noticed that some students, like Maya, were able to claim that they disagreed with Ms. Sanchez and then prove their claim was true by using a counterexample; other students, like Oscar, had more difficulty determining if Ms. Sanchez's claim was true or false. Ms. Sanchez realized that students like Oscar would need some more support with this concept.

The students in Ms. Sanchez's class were writing mathematically to construct an argument. A mathematical argument is "a sequence of statements and reasons presented with the aim of demonstrating that a claim is true or false" (Casa et al., 2016, p. 13). In this vignette, the students were presented with the claim that the whole of one item is always larger than the half of another item and were challenged with writing an argument to communicate their reasoning.

Toulmin (1958) described the structure of an argument as having a claim, evidence to support the claim, and a warrant to connect the evidence to the claim. For example, Maya's claim is that she disagrees with the idea that the whole is always larger. Her evidence is a comparison between half an ocean and a puddle. The warrant of her argument is that the claim depends on the size of the whole. Whereas Maya wrote out her argument using only text, other students may have chosen to include visual representations to support their ideas.

When engaged in argumentative writing, students are writing to an audience beyond themselves, whether another student, teacher, or someone outside the classroom. In this vignette, Ms. Sanchez is the audience for the students' writing because she has presented students with this particular claim and will be reading their work. Because the students are writing to an outside audience, precision and clarity are highly valued in this type of writing. The level of sophistication of the argument and critique will depend on the developmental level of the students and their current understanding of the concept.

In addition to writing mathematically to construct an argument, students engaged in argumentative writing might also write mathematically to critique an argument posed by someone else. In writing to critique an argument, a student must first understand the argument that is presented by another and then decide how to respond to it. For instance, Ms. Sanchez may decide to have her students critique each other's arguments. In Oscar's case, having someone critique his argument may help extend his understanding of a whole in relation to the fractional size. For both purposes of argumentative writing, students must select the evidence and warrants that will most effectively support the construction or critique of an argument.

Mathematically Creative Writing

> Anita, a student in Ms. Sanchez's classroom, was really interested in the fraction discussions. In fact, she had also been thinking about fractions as decimals, and as she worked on a math worksheet in class, she had an idea. Anita used extra space on her worksheet to write her problem: "If one third as a decimal is 0.333333... and two thirds = 0.66666..., then why is three thirds equal to 1 and not 0.9999999...?" Ms. Sanchez was excited to see Anita's work and spent some extra time with Anita to discuss the problem she posed. Anita's writing helped advance her understanding of the relation between decimals and fractions.

Anita's writing is an example of mathematically creative writing, which, like exploratory writing, is often student initiated. Mathematically creative writing engages students in documenting original ideas, problems, and/or solutions, conveying fluency and flexibility in thinking, and elaborating on ideas (Casa et al., 2016). It is characterized by students going beyond the teacher's intended or expected outcome of the task, problem, or situation.

In this vignette, Anita went beyond her class's learning of fractions and posed a problem. Problem posing can encourage students to think of innovative mathematical ideas by questioning mathematical concepts. For instance, students learning about two-digit subtraction may notice that in a problem such as 32 – 17, they can first subtract 2 − 7 to find a difference of −5, next subtract 30 − 10 to find a difference of 20, and then subtract the five 1s from 20 or add the −5 to 20 to solve this problem.

Although Anita's nontraditional approach is not a new idea in mathematics, this would be an original thought for a student if his or her teacher had yet to teach this concept. This approach also demonstrates the student's fluency and flexibility in mathematical thinking by realizing properties of subtraction. Mathematically creative writing can provide students with an outlet to communicate ideas that go beyond the parameters of a given task. Students may decide to share this writing with others or may keep it to themselves; hence, the audience for mathematically creative writing can vary.

Strategies to Engage Students in Mathematical Writing

Equipped with a clearer vision of the types of and purposes for mathematical writing, teachers are well positioned for the next step: engaging students in mathematical writing in the classroom. So what strategies can help accomplish this goal? There are resources, such as *5 Practices for Orchestrating Productive Mathematics Discussions* (Smith & Stein, 2011), *Academically*

Productive Talk: Supporting Students' Learning in Mathematics (Chapin & O'Connor, 2007), and *Intentional Talk: How to Structure and Lead Productive Mathematical Discussions* (Kazemi & Hintz, 2014), that can provide teachers like Ms. Sanchez with support for engaging students in oral discourse specific to mathematics.

However, the task force noted that there are few resources available to support elementary teachers with the implementation of writing in mathematics class, and virtually none aimed specifically at each of the four types of and varied purposes for mathematical writing (Casa & Firmender, 2015). Those available, such as *Writing in Math Class* (Burns, 1995) and *Writing to Learn Mathematics* (Countryman, 1992), were written before current mathematics standards initiatives (e.g., NGA Center & CCSSO, 2010b; NCTM, 2000) and were not specifically written to support teachers with reasoning-based mathematical writing because this distinction had yet to be made.

Although identifying strategies for engaging students in each specific type of mathematical writing has yet to be done, we can look more broadly to what we know about writing and oral discourse, both generally and specifically in mathematics, to provide some strategies that can assist with the integration of elementary mathematical writing into mathematics instruction. Consider how Ms. Sanchez has incorporated strategies to engage her students in writing to learn about fractions.

> Ms. Sanchez started the new school year excited to apply what she learned about engaging students in mathematical writing. As she posed the same problem, asking why one half is greater than one fourth, to her new group of students, she reflected how she could use writing to support her students' mathematical reasoning about fractions.
>
> This time, instead of just explaining orally, Ms. Sanchez used words and pictures to provide a visual representation of her ideas. First, she drew circles of the same size and divided one into two equal parts and the other into four. Then she wrote "I know one half is greater than one fourth because I have two circles of the same size, and I divided one circle into two equal parts and the other into four equal parts, I can see that one part of the circle divided into two parts is larger than one part of the circle divided into four parts." While writing, she shared her ideas out loud to model her thinking for her students and to explain how she crafted her message.
>
> Next, she posed a similar problem, "Why is one third less than one half?" This time, she asked students to work with partners, first talking about their ideas, then using writing to communicate their ideas. With chart paper, she provided a prompt to help students get started: "I

know one third is less than one half because ___." She observed some students drawing circles and dividing them into parts while others grabbed manipulatives from the nearby supply cart. She listened as partner groups talked about and wrote their ideas, noting students who seemed to grasp the concept and providing feedback to help guide those who had some confusion. As student groups shared, each student in the class saw how others used writing to communicate their ideas.

Finally, Ms. Sanchez had each student complete an exit ticket that prompted "Which is greater, two thirds or three fourths? How do you know?" From her students' responses, Ms. Sanchez could not only see whether students could identify which was greater, but she could also gain insight into each student's understanding and reasoning.

In this vignette, Ms. Sanchez incorporated several strategies to support her students' mathematical writing. For example, she integrated both teacher and peer modeling to scaffold the writing process for her students. She provided students with ongoing practice and feedback, providing prompts and support when needed, gradually building students' independence. Ms. Sanchez also provided explicit opportunities for students to engage in oral discourse, using it as a springboard into the writing.

As Ms. Sanchez continues to integrate mathematical writing into her lessons, an important next step is to integrate rich and worthwhile mathematics tasks (NCTM, 2000) that encourage reasoning and problem solving (NCTM, 2014). Such tasks have multiple solution paths (2014) and have the potential to engage students in higher level thinking, mathematical discussions, and writing (Colonnese, 2017). Smith and Stein (1998) provided this example: "Create a real-world situation for the following problem: ⅔ × ¾. Solve the problem you have created without using the [traditional algorithm for multiplying fractions] and explain your solution" (p. 349).

This task is grounded in an authentic context that provides a meaningful opportunity for students to access relevant knowledge and experiences. It creates a higher level cognitive demand for students by challenging them to explore mathematical relations (e.g., multiplying fractions in nonalgorithmic ways and conceptualizing the whole as ¾ to obtain ⅔ of that amount) and requires significant effort to apply conceptual understanding while working through task constraints such as the limitation of not using the algorithm for multiplying fractions (Smith & Stein, 1998). When crafted with an eye toward incorporating mathematical writing, rich and

worthwhile tasks can enhance students' abilities, such as documenting problem-solving processes, forming generalizations, and drawing connections to previously learned concepts.

Conclusion

Writing can be a powerful tool that prompts students to think deeply and illuminates for teachers the depth and quality of thinking happening in a student's mind. The more students and teachers can share and reflect on written ideas, the more deeply students may internalize concepts fundamental to mathematical content. Engaging students in mathematical writing both extends students' communication skills and leverages the power of writing to enhance learning within the discipline of mathematics. Understanding the types of and purposes for writing in the context of mathematics as outlined by the task force, and how those types and purposes can frame the use of mathematical writing in the elementary classroom, is an important step toward the goal of supporting students' discipline specific writing.

TAKE ACTION!

1. Provide and make accessible writing materials to students during mathematics class.
2. Develop writing tasks for students that will engage them in writing to reason and to communicate mathematically.
3. Model all types of mathematical writing for students.
4. Ask students to write for a variety of purposes.
5. Provide time for students to write about their mathematical ideas.
6. Encourage students to use exploratory writing to conjecture and mathematically creative writing to document an original idea.
7. Offer supports for students who may be struggling to write such as sentence starters or paired writing opportunities.
8. Share exemplars and samples of writing to help students understand how to write for different purposes.
9. Provide students with an authentic audience.
10. Celebrate students' writing!

Notes

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MORE TO EXPLORE

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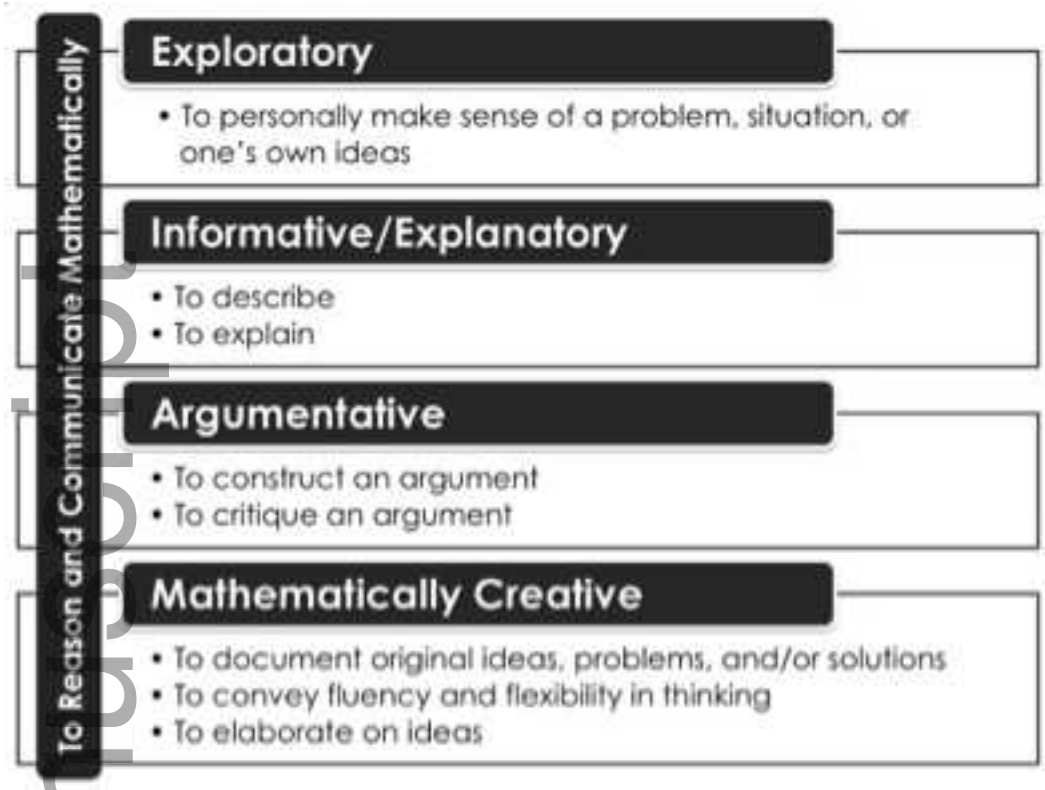
• Visit the Elementary Mathematics Writing Task Force webpage (https://mathwriting.education.uconn.edu/) for more information on the recommendations and resources.

Figure 1.

The recommended overarching goals of, types of, and purposes for elementary mathematical writing

Note. From *Types of and Purposes for Elementary Mathematical Writing: Task Force Recommendations* (p. 4), by T.M. Casa, J.M. Firmender, J. Cahill, F. Cardetti, J.M. Choppin, J. Cohen, … R. Zawodniak, 2016, retrieved from http://mathwriting.education.uconn.edu/resources-for-elementary-mathematical-writing/. The report is in the public domain.



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